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Salt fingers in the presence of grid-generated turbulence

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The transport of heat and salt across a density interface containing salt fingers is investigated when turbulence produced by vertically oscillating horizontal grids is imposed on the deeper layers above and below the interface. The fluxes of heat and salt are measured as functions of the stirring frequency. The results are discussed with reference to the parameter $\lambda = w^2/(\overline{u})^2$, where w is the velocity of the fluid in the fingers in the undisturbed state and $\overline{u}$ is the r.m.s. horizontal velocity of the turbulence. It is found that the salt flux has a minimum (as a function of the stirring frequency) when $\lambda \sim 0{\cdot}3$ and that when $\lambda \lesssim 0{\cdot}05$ the transport across the interface is dominated by mechanical mixing. The ratio r of the contributions of heat and salt to the buoyancy flux increases with decreasing λ and $r > 1$ when $\lambda \lesssim 1{\cdot}3$. The latter result implies that if turbulent intensities are such that $\lambda \lesssim 1{\cdot}3$ in a particular oceanic situation, mechanical mixing will affect the vertical transport of heat and salt in such a way that the salt finger mechanism will be unable to produce layered temperature and salinity microstructure in the manner described by Turner (1967).

1. Introduction

The possible importance of salt finger convection as a means of transport of heat and salt in the ocean has been pointed out by various authors (e.g. Stern 1967). An important property of salt fingers is their ability to produce layered temperature and salinity structure from smooth gradients. Turner (1967) has shown, in the laboratory, that layers can be formed by imposing an unstable salt flux at the top of a stable temperature gradient. Layer formation has also been demonstrated by Stern & Turner (1969) using sugar and salt. The layered structure found below the Mediterranean outflow has been attributed to salt fingers (Tait & Howe 1968); the essential feature of this layer formation is that the buoyancy flux through the fingers due to salt F_S is greater than the corresponding heat flux F_H.

Turner (1968) has shown that turbulence generated by an oscillating grid has a significant effect on the separate transports of heat and salt across a density interface. Thus the possibility exists that salt finger convection may be altered by externally imposed turbulence. In order to determine the nature of salt finger convection when exposed to grid-generated turbulence some experiments have been carried out in a tank containing stirring grids. The results of these experiments have been interpreted using information about the structure of the turbulence generated by an oscillating grid obtained by Thompson (1969). This information allows the results of the experiments described in this paper to be related to the detailed properties of the salt finger motions and the imposed turbulence. Further, some conclusions are drawn on the possible importance of salt fingers in the ocean.

2. The experimental method

The experiments were conducted in the tank shown in figure 1. Two grids, of 1 cm square Perspex rods made into a 5×5 array with 5 cm between the centres, were mounted on a rod passing vertically through the middle of the tank. A rule was fastened to one wall of the tank and fitting neatly between two grid bars

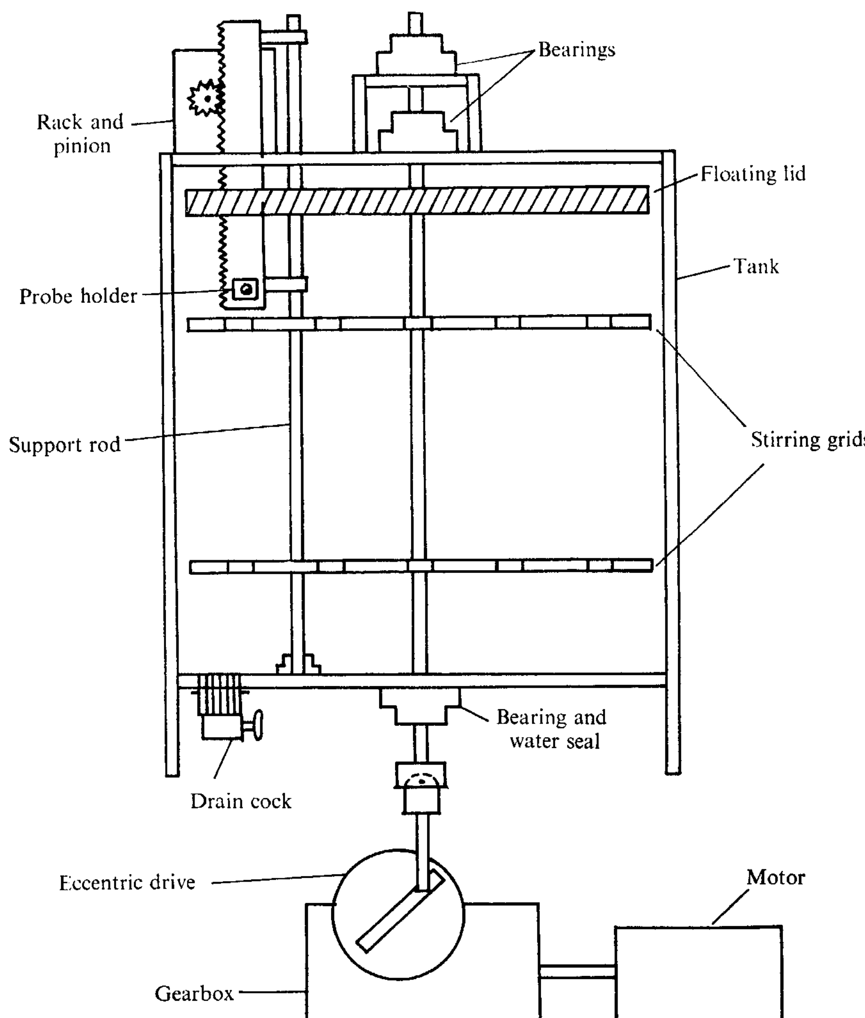

FIGURE 1. A sketch of the experimental tank showing the rack and pinion drive used to obtain profiles of temperature against depth.

prevented them from rotating. The central rod was connected via a crank to an electric motor and the frequency of oscillation of the grids was adjusted to a number of discrete frequencies by means of a stroboscopic disk mounted on the driving wheel.

Each experiment was set up by first placing a layer of warm water in the tank and then slowly introducing a layer of cooler water underneath. In every case both layers were of equal depth (18 cm). The grids, which were located at the centre of each layer, were then oscillated for approximately 5 min in order to

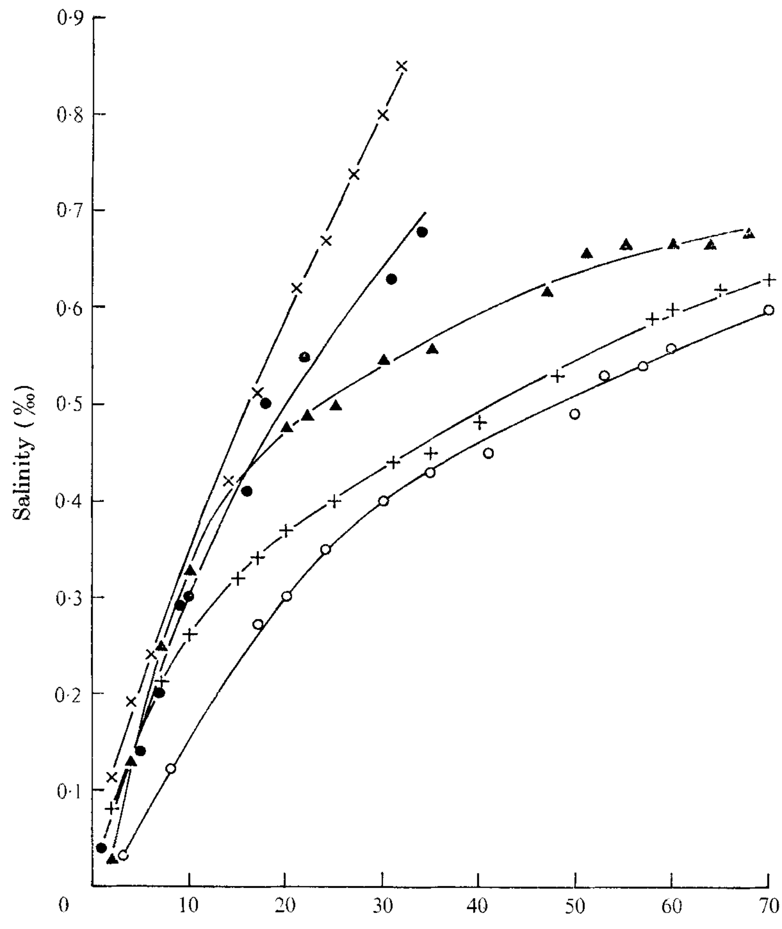

FIGURE 2. A plot of the increase in salinity of the lower layer against time for various values of the stirring frequency. All the runs shown have the same initial conditions, viz. $\Delta T = 7{\cdot}0$ ‰, $\Delta S = 1{\cdot}8$ ‰ ●, 0 c/s; ▲, 1·1 c/s; ○, 2·2 c/s; +, 2·8 c/s; ×, 5·0 c/s.

sharpen the interface and allow the velocity field to reach a steady state (Thompson 1969). At this stage a known amount of salt dissolved in hot water was added through an expanded polystyrene sheet floating on the water surface. A stop watch was started when the salt was added, and measurements of temperature and salinity were taken at discrete times during a run.

The temperature was measured by thermistors calibrated to 0·05 °C over the range 15–60 °C. The thermistors were connected via a bridge circuit to an $X-Y$ plotter. The salinity was determined by using a single electrode (platinum) conductivity probe in an a.c. bridge. The probe was calibrated directly against temperature and salinity and was accurate to 0·01 ‰.

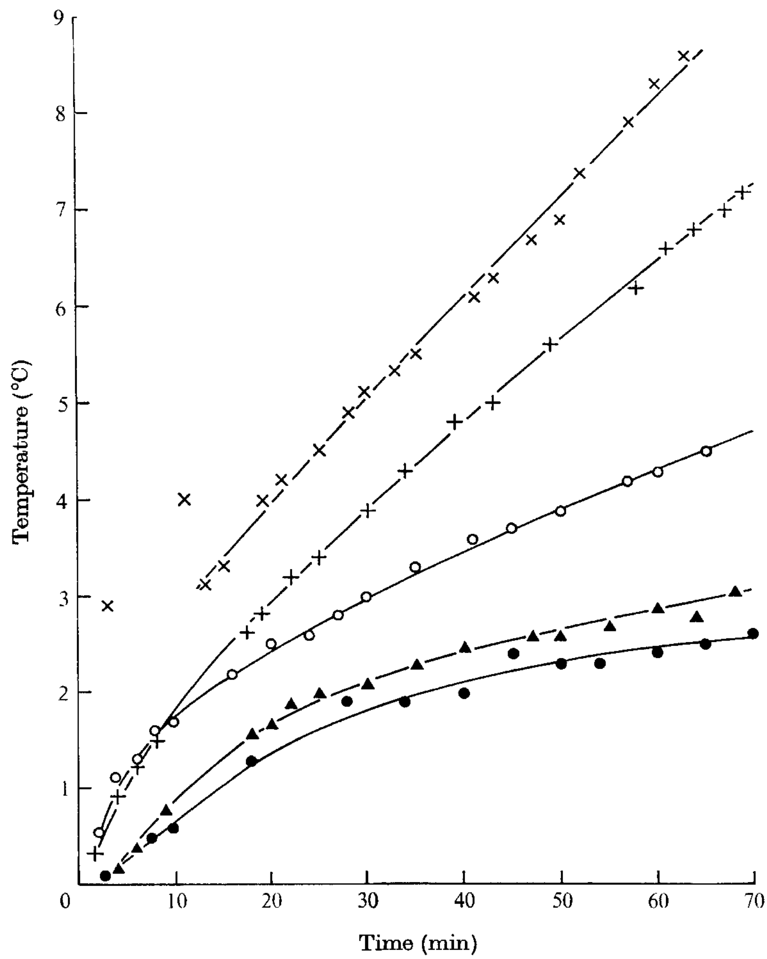

FIGURE 3. The increase in temperature in the lower layer corresponding to the salinity increases shown on figure 2. The symbols have the same meaning as those on figure 2.

In order to relate the properties of the interface to the imposed turbulence profiles of temperature against depth were taken. These profiles were used as a measure of the depth of the region containing salt fingers. The profiles were obtained by connecting a thermistor to a rack and pinion drive, the depth being recorded by the output of a potentiometer attached to the pinion shaft. Profiles were then made directly by recording the thermistor and potentiometer outputs on the two axes of the $X-Y$ plotter.

3. Experimental results

Variations of heat and salt flux with stirring frequency

The fluxes of heat and salt were determined from measurements of the temperature and salinity of the lower layer with increasing time. The temperature measurements were corrected for heat losses through the walls of the tank. The stirring frequency was varied over the range 0–5 cycles per second.

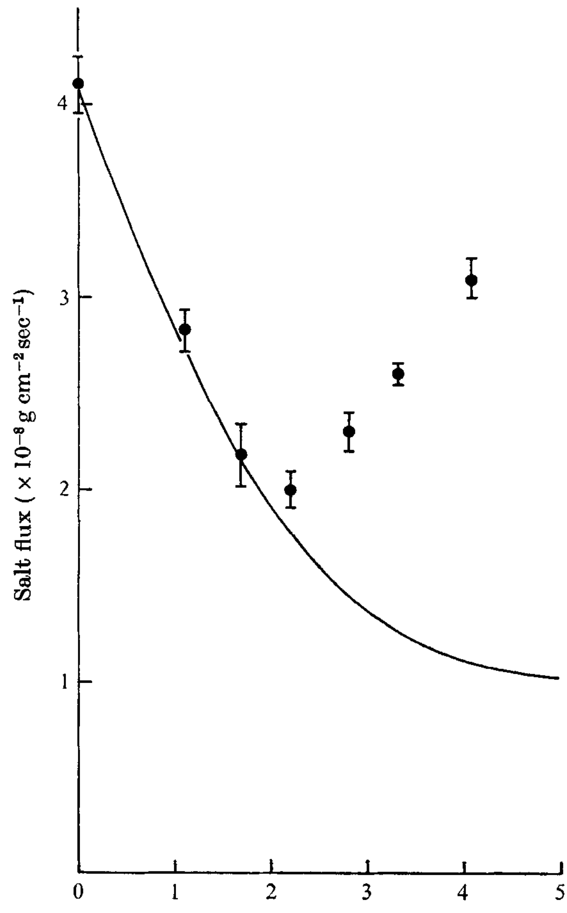

FIGURE 4. The salt flux, averaged over 70 min with initial values of $\Delta S = 1{\cdot}8\,‰$ and $\Delta T = 7{\cdot}0\,‰$, as a function of the stirring frequency. Plotted for comparison is the curve given by (5.1) and (5.2). The error bars denote the magnitude of the standard deviation from the mean.

In order to compare the fluxes for different stirring frequencies a series of runs was carried out using the same initial conditions; viz. $\Delta S = 1{\cdot}8‰$ and $\Delta T = 7{\cdot}0‰$ where ΔS and ΔT are the salinity and temperature differences (measured in buoyancy units), respectively, between the upper and lower layers. Figure 2 shows the increase in salt content of the lower layer as a function of time for a selection of stirring frequencies. It can be seen that the salt flux has a minimum, as a function of frequency, when the stirring frequency is 2·2 c/s. The averaged value of the salt flux at this minimum is $2{\cdot}0 \times 10^{-8}\,\mathrm{g\,cm^{-2}\,sec^{-1}}$. The increase in the temperature of the lower layer is shown on figure 3. In contrast to the salt flux, the heat flux is an increasing function of the stirring rate over the range covered in these experiments. The values of the salt and heat fluxes are shown on figures 4 and 5, respectively. These values were obtained by determining the increase in salinity per unit time averaged over the time for which the ratio of the heat and salt fluxes remains constant (see figure 6).

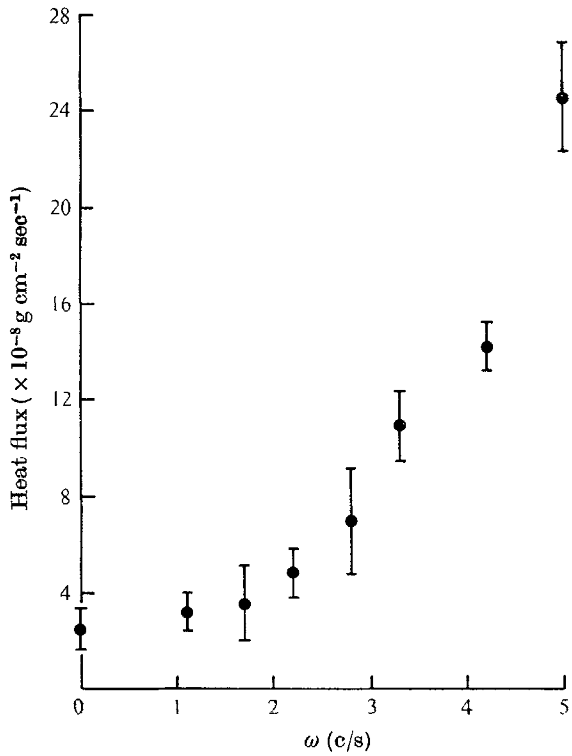

FIGURE 5. The heat flux, averaged over 70 minutes with initial values of $\Delta S = 1{\cdot}8\,‰$ and $\Delta T = 7{\cdot}0\,‰$ as a function of the stirring frequency.

Figure 6 shows typical graphs obtained when the increase in temperature of the lower layer is plotted against the corresponding increase in salinity for a series of runs with different initial conditions. These plots show that, at each frequency, the ratio of the buoyancy fluxes $r = F_H/F_S$ (given by the slopes of the lines) is constant for values of $\Delta T/\Delta S$ from 1·5–5·0. (The values of $\Delta T/\Delta S$ are indicated by the numbers at the ends of each line.) The points above the lines of constant r on figure 6 are thought to be due to mixing produced by the turbulence when the interface stability becomes small. The ratios of the buoyancy fluxes are plotted against frequency on figure 7. Each value is the average of at least three runs. It should be noted that r tends to a constant value at the higher frequencies. The value of this limit is the initial value of $\Delta T/\Delta S$ (see, for example, figure 6(*d*));

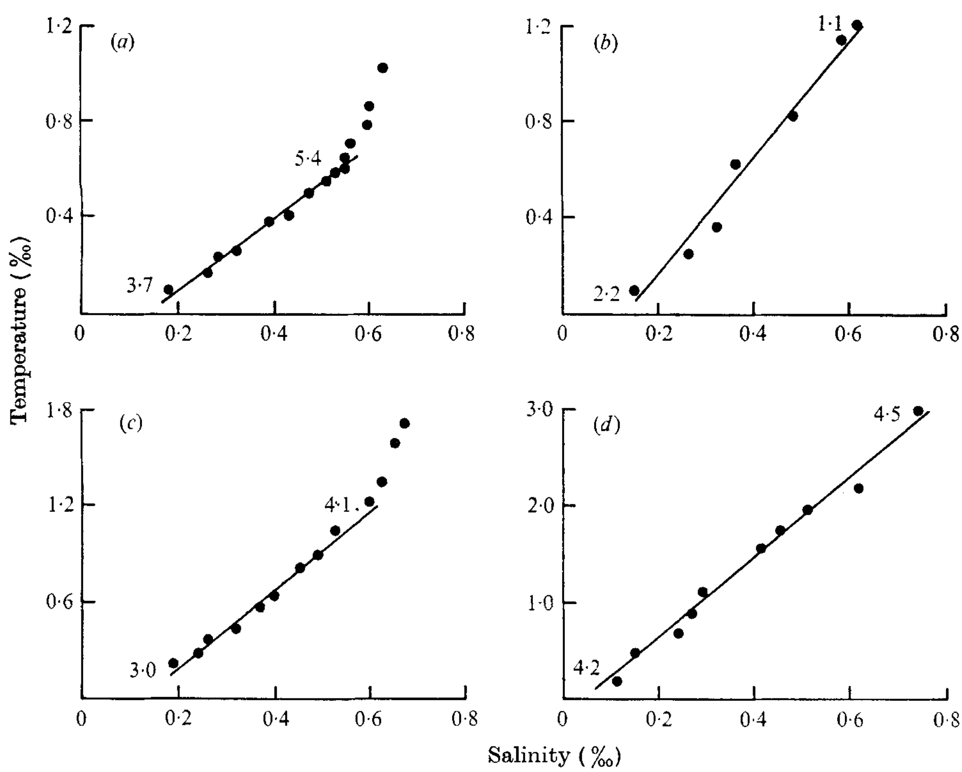

FIGURE 6. The change in temperature of the lower layer as a function of the corresponding change in salinity. The values of $\Delta T/\Delta S$ are given by the numbers at the ends of the lines. (*a*) $\omega = 1\cdot7$ c/s, (*b*) $\omega = 2\cdot2$ c/s, (*c*) $\omega = 2\cdot8$ c/s, (*d*) $\omega = 5\cdot0$ c/s.

this feature indicates that the transport is due to mixing produced by the turbulence. The values shown on figure 7 represent the total transports of heat and salt across the interface. No attempt has been made to separate the heat flux due to the salt fingers from that produced by conduction or by the turbulence acting on the interface for the following reasons. First, in the ocean both types of transport are present, and second, it is not clear on what basis such a separation can be made. In the case when $\Delta T/\Delta S$ is large, the heat flux due to the fingers is much smaller than that produced by the action of the turbulence: when $\Delta T/\Delta S \sim 1$, the density step between the layers is reduced by the presence of the salt thereby increasing the turbulent heat flux.

Interface thickness

Profiles of temperature against depth were taken in order to determine how the thickness of the interface containing the salt fingers varies with the frequency of oscillation of the grids. These profiles were obtained by lowering a thermistor at about $0\cdot5$ cm sec^{-1} through the interface. The response time of the thermistor was approximately $0\cdot1$ sec and the profiles represent an instantaneous picture of the vertical temperature field at one horizontal position in the tank. A typical

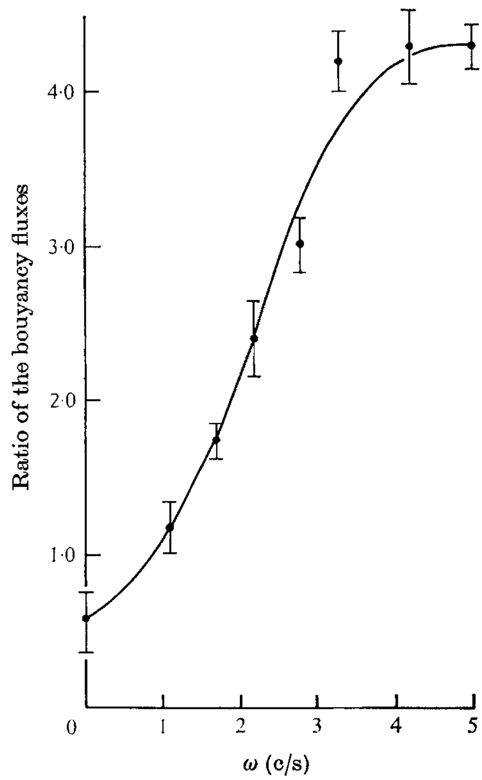

FIGURE 7. The ratio of the bouyancy fluxes, calculated from the slopes of lines such as those shown on figure 6, as a function of the stirring frequency.

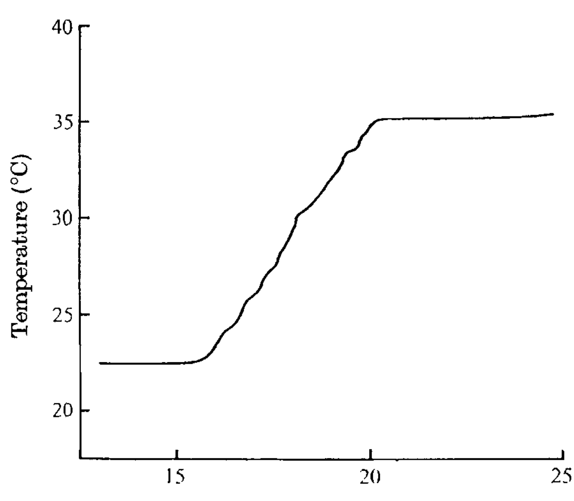

FIGURE 8. A typical temperature-depth profile showing the linear temperature gradient through the fingers.

profile is shown on figure 8. The linear temperature gradient through the interface was a feature common to all the profiles. Temperature inversions were observed in some of the profiles. In every case these inversions occurred at the edges of the gradient region and were presumably a result of overturning produced by the action of a turbulent eddy. No inversions were found in the interior of the gradient region.

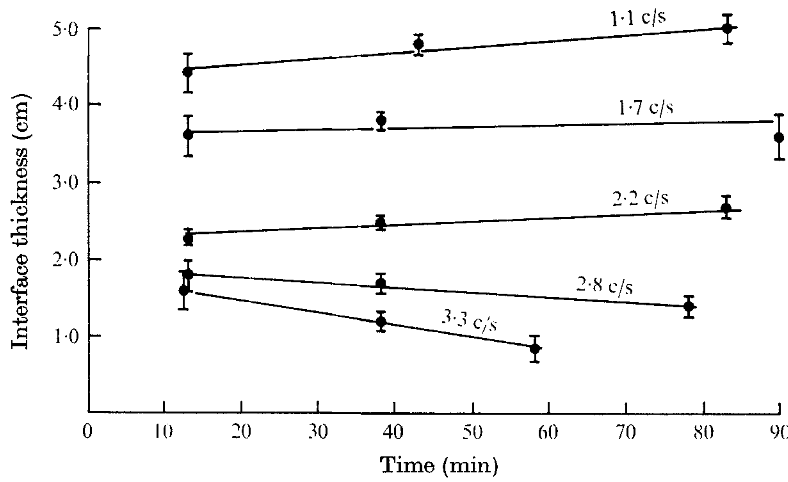

FIGURE 9. Values of the interface thickness, taken from temperature-depth profiles, for a series of runs with the same initial conditions but with different stirring frequencies.

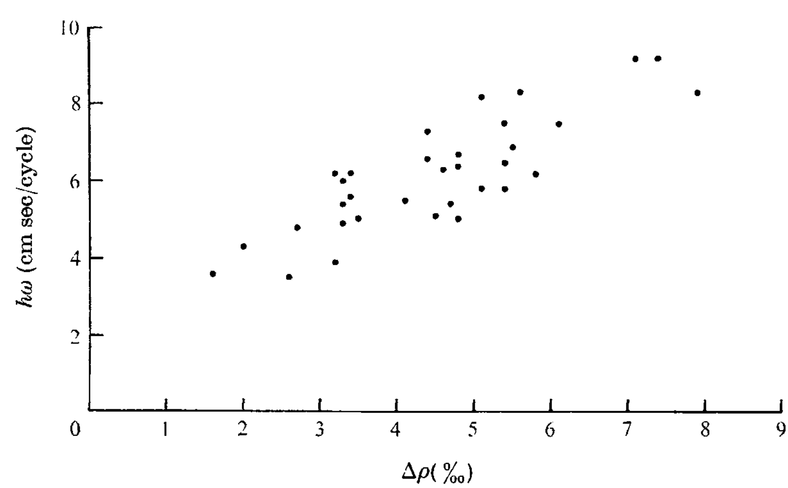

FIGURE 10. A plot of the interface thickness multiplied by the frequency (cm sec/cycle) against the density difference between the layers ($\times 10^3\,\mathrm{g\,ml^{-1}}$).

Measurements of the interface thickness were made at discrete times during a run. The results of these measurements are shown on figure 9. For frequencies less than 2·2 c/s the interface thickness increases with time, but at higher frequencies the interface becomes thinner during a run. The change in width of the interface as the stirring frequency is increased is also on figure 9. In order to determine the relationship between the stirring frequency ω, the interface thickness h and the salinity and temperature differences between the layers, profiles were taken for varying conditions. The results of these measurements are shown on figure 10; the frequency was varied over the range 1–3·3 c/s and the values of $\Delta T/\Delta S$ from 2 to 6. To within experimental accuracy, the best fit for the data was

$$h\omega = K\Delta\rho, \tag{3.1}$$

where $\Delta\rho = \Delta T - \Delta S$ is the density difference between the layers and

$$K = 0{\cdot}1 \pm 0{\cdot}02\,\text{g}^{-1}\,\text{cm}^5\,\text{sec}^{-1}.$$

4. Some theoretical preliminaries

In order to relate the results described above to the detailed properties of the salt fingers and the turbulence it is necessary to discuss briefly some features of salt finger convection in the absence of any externally imposed turbulence. Assume (as is the case in the experiments) that the salt fingers exist in a thin interface between two deeper well-mixed layers. Suppose the motion in the fingers is vertical; then the steady equations of momentum, heat and salt conservation (to the Boussinesq approximation) are

$$0 = -g(\rho-\rho_0)/\rho_0 + \nu\nabla_H^2 w, \tag{4.1}$$

$$w\,T_z = \kappa_T\nabla_H^2 T, \tag{4.2}$$

$$wS_z = \kappa_S\nabla_H^2 S, \tag{4.3}$$

where w is the vertical velocity of the fluid in the fingers, g is the acceleration due to gravity and T and S are the temperature and salinity, respectively. The density ρ is related to the temperature and salinity by the equation

$$\rho = \rho_0(1-T+S). \tag{4.4}$$

For the sake of convenience the coefficients of expansion for heat and salt are assumed to be constant; the values of these constants are taken as unity using appropriate units for T and S. Following the observations of Shirtcliffe & Turner (1970) we assume that the salt fingers have a square plan form. Then these equations have solutions of the form

$$(w, T, s) = (0, \overline{T}(z), \overline{S}(z)) + (\hat{w}, \hat{T}, \hat{S})\sin(\pi x/L)\sin(\pi y/L); \tag{4.5}$$

L represents the width of a salt finger and a bar denotes a horizontal average. Define

$$r = F_H/F_S, \tag{4.6}$$

to be the ratio of the buoyancy fluxes of heat and salt across the salt finger interface. Then (4.1)–(4.6) give (after some simple manipulations)

$$\hat{w} = gL^2\hat{S}(r-1)/2\nu\pi^2. \tag{4.7}$$

The salinity step ΔS between the upper and lower layers is taken up in two transition regions at the ends of the fingers ($\overline{S}_z/\overline{T}_z \sim 10^{-2}$ through the finger region). Therefore it will be assumed that $\hat{S} \sim \frac{1}{2}\Delta S$. Turner (1967) found that in the absence of stirring the value of r was a constant; the numerical value of this constant; 0·56, was confirmed in these experiments (see figure 7). Further, a shadowgraph showed that a typical horizontal length scale for the fingers is 0·2 cm. These values imply that for the conditions used in the runs shown on figure 2 a typical value of the velocity of the fluid in the fingers is 0·07 cm sec^{-1}.

Thompson (1969) measured the r.m.s. horizontal velocity $\overline{u}$ and integral length scale of the turbulence generated by the grid in a homogeneous fluid. His measurements, made with a hot-film probe, showed that the velocity and length scale of the turbulence varied with the frequency and amplitude of oscillation of the grid, the distance from the grid and the shape of the grid. For the particular geometry used in the experiments reported here his measurements show that r.m.s. horizontal velocity of the tubulence generated at the interface by the grid is given by

$$\overline{u} = 0{\cdot}06\omega \,\mathrm{cm\,sec^{-1}}, \tag{4.8}$$

where ω(c/s) is the frequency of oscillation of the grids. The integral length scale of the turbulence is approximately 1 cm. Although Thompson's measurements were made in a homogeneous fluid (4.8) is used as a measure of the imposed turbulence in the homogeneous layers on either side of the salt finger interface. The effects of the density gradient on the turbulent motions is not considered here. The comparison of the results of the mixing produced by one or two grids in a one component fluid obtained by Turner (1968) indicates that the two sides of the interface are decoupled in the sense that the simultaneous arrival of two eddies on different sides of the interface at the same position is a rare event. Therefore, we may take $\overline{u}$ as a measure of the velocity difference across the interface and it is now possible to use (4.8) to evaluate the Richardson number defined by

$$Ri = (g\Delta\rho/\rho_0)h/(\overline{u})^2. \tag{4.9}$$

However, the intensity of the salt finger convection also depends on the ratio $\Delta T/\Delta S$. A more satisfactory measure of the effect of the turbulence is given by the parameter

$$\lambda = w^2/(\overline{u})^2, \tag{4.10}$$

where again $\overline{u}$ is taken as representative of an average velocity difference across the interface. λ is related to Ri in the following way. In the absence of stirring Turner (1967) has shown that

$$F_S \propto f(\Delta T/\Delta S)\,(\Delta S)^{\frac{4}{3}}, \tag{4.11}$$

where f is a slowly-varying function of $\Delta T/\Delta S$. Also Stern & Turner (1969) show that

$$F_S \propto w\Delta S,$$
$$\propto \Delta S/h.$$

Therefore

$$\lambda \propto f^3\left(\frac{\Delta T}{\Delta S}\right)\Delta Sh/(\overline{u})^2,$$
$$\propto f^3\left(\frac{\Delta T}{\Delta S}\right)Ri\bigg/\left(\frac{\Delta T}{\Delta S}-1\right). \tag{4.12}$$

5. Discussion of the results

It is helpful to use the results obtained from experiments with salt fingers in the absence of externally imposed turbulence as a reference against which comparisons are made. Then the theme which characterizes the results is the disruption of the ordered salt finger motions by the turbulence as the stirring frequency increases until, finally, all the transport across the interface is produced by mechanical mixing.

The minimum value of the salt flux as shown on figures 2 and 4 is consistent with this disruptive effect. If it is assumed that the horizontal shear associated with the turbulence rotates the fingers through an angle θ (on the average), then the reduced vertical velocity of a parcel of fluid in a finger implies a reduced salt flux given by

$$F_S = F_S^0 \cos\theta, \tag{5.1}$$

where the superscript 0 denotes the magnitude of the salt flux at zero stirring frequency. Now, provided that (as is the case in these experiments) the length scale of the turbulent motions is of the same order as or larger than the interface thickness,

$$\cos\theta = (1+1/\lambda)^{-\frac{1}{2}}. \tag{5.2}$$

The curve given by (5.1) and (5.2), using the value of w from (4.7) is shown in figure 4. The experimental values of the salt flux agree with this calculation for frequencies less than 2·2 c/s. The increased salt flux at higher stirring rates is caused by mechanical mixing.

The fact that the heat flux is an increasing function of the stirring frequency is not a contradiction of the idea stated above. The fingers are shortened by the turbulence (figure 10) and therefore the temperature gradient across the interface is increased thereby giving a larger heat flux. (The corresponding effect is not found in the salt flux as most of the salinity difference between the layers is taken up in two thin transition regions at the ends of the fingers.) Further, figure 5 shows that the heat flux increases more rapidly for $\omega > 2{\cdot}2$ c/s than it does for the lower frequencies. This change also indicates that the turbulence is becoming the dominant process in the transport of heat across the interface at these higher frequencies.

The gradual change from an interface controlled by salt finger convection to one at which most of the transport occurs by mechanical mixing is also apparent in the measurements of the interface thickness against time shown on figure 9. It has been demonstrated by Stern & Turner (1969) that the width of a salt finger interface increases linearly with time. This trend is noticeable for frequencies up to 2·2 c/s. For larger frequencies, the interface becomes thinner with increasing time, a result of the turbulent eddies sweeping away the edges of the salt fingers. The rapid increase of r over the frequency range 0–4 c/s is due to the combined effects of an increasing heat flux and a decreasing salt flux. At higher frequencies r approaches a constant. As was mentioned above the latter implies that the fingers are completely disrupted by the turbulent eddies and have no significant effect on the transports at these frequencies.

6. Summary and conclusions

The experiments reported here indicate three main features of the transport of heat and salt across a salt finger interface with externally imposed turbulence. First, the ratio of the buoyancy fluxes becomes greater than unity for $\omega \gtrsim 1$ c/s; second, the salt flux has a minimum at $\omega \sim 2{\cdot}2$ c/s and third, for $\omega \sim 5$ c/s the transport is dominated by mechanical mixing. These results may be expressed in terms of the Richardson number of the flow by means of (4.8) and (4.9). However, the values of the Richardson number so obtained are special in the sense that they refer to the particular geometry used in these experiments. Further, the Richardson number does not uniquely specify the flow as it does not take into account the value of $\Delta T/\Delta S$. The results of these experiments indicate that λ is a more satisfactory parameter for determining the relative importance of the salt finger and turbulent fluxes.

The ratio of the buoyancy fluxes is greater than unity for all the stirring frequencies (except zero) available. Thus, even for very large Richardson numbers $\sim 10^3$ ($\lambda \lesssim 1{\cdot}3$), the density step between two layers will decrease with increasing time and will eventually disappear. Furthermore, the mechanism proposed by Turner (1967) for producing layered temperature and salinity structure from smooth gradients by imposing a downward salt flux at the top of a stable temperature gradient relies on r being less than one. It is therefore necessary to have detailed velocity measurements before it can be concluded that salt fingers are responsible for some of the layering observed in the temperature and salinity fine structure in the ocean.

The minimum value of the salt flux occurs when $\lambda \sim 0{\cdot}3$ ($Ri \sim 130$) and complete mixing, as indicated on figure 6(*d*), when $\lambda \lesssim 0{\cdot}05$ ($Ri \lesssim 30$). However, oceanic measurements of turbulence are not yet available for comparison. Measurements made by Grant, Molliet & Vogel (1968) of the turbulence in and above the thermocline near Vancouver Island indicate that below about 200 m the turbulence is patchy, with approximately 10 % of the water being turbulent at any one time. Thus even though the fingers may be disrupted by turbulent motions locally they could still have a significant effect on the vertical transports in the ocean. These experiments have not taken into account the effect of the vertical gradient of mean horizontal velocities. The effects of steady mean shears on salt finger convection is at present under investigation.

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