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Mixing Processes in a Highly Stratified River

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Abstract

Several of the mixing processes at density interfaces in highly stratified flow at the mouths of rivers flowing into the ocean bear close resemblance to mixing processes at the density interface seen in freshwater bodies, and can provide valuable insights into them. This paper addresses the mixing phenomena at the interface in the very highly stratified flow seen at the mouth of the Ishikari River during the dry season. It is shown that the chief factor influencing mixing across the interface is the wind, while factors disturbing the steadiness of the flow such as tide have little influence. Furthermore, when the entrainment coefficient is examined as a function of wind speed, it is found that the surface salinity at any given river station can be accurately predicted. Mixing mechanisms are also discussed.

Introduction

Commonality Between Limnology and Flows in Highly Stratified Flows at River Mouths

The physical processes of stratification and mixing take place between sea water and fresh water at the mouth of a river. When the volume flow rate is extremely high, as in the case of the Amazon River, or when the level of the river at the mouth is always higher than the level of the highest tide, the salt wedge never enters the river, and the processes all take place in the ocean; these are outside the concerns of limnology. However, the levels of many rivers are below the mean sea level at their mouths and their flow rates are not always high enough to flush the denser seawater from the riverbeds. As a result, sea water is sometimes included in the water drawn from rivers or lagoons for agricultural uses. In highly stratified flows, which are particularly common when the tidal range is small, it is not at all rare to observe salt intrusions of several tens of kilometers inland. Intrusions of 200 km have been documented in the Mississippi, which flows through an extremely flat plain in its lower reaches. Thus, the above processes may be observed between sea water and fresh water even at locations distant from the sea, whether in rivers or in wetlands, and relate directly to limnology. The processes of mixing which takes place in highly stratified flow at river mouths are qualitatively identical to processes of limnological stratified flows; for example, underflows, flow at the base of the surface layer and flow at the top of the benthic boundary layer. This study addresses several aspects of highly stratified flow at the mouth of a river which flows into a body of

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comparitively low tidal range with a view to the insights they provide into current topics in limnology.

Review of Research on Mixing of River Water and Sea Water in Estuaries and River Mouths

Invasion of ocean water into rivers or estuaries has long posed problems in humankind's use of water resources. The Dutch complain of corrosion by sea water in records dating from the 1300's. The same set of problems beset man all over the world, and countries of Europe and North America have been tackling the practical challenge of salt invasion for over one hundred years (Proc. ICN, 1953). Therefore, research in the West has a long history, with river mouth flows generally classed into cases of estuary flows. The most significant findings are presented by Prichard (1963), Ippen (1966), Dyer (1972) and Officer (1976). Prichard classified river mouth flows into highly stratified (salt wedge), partially mixed and homogeneous (well mixed) flows. Because of the actual cases most often seen by Western researchers, their writings have focussed mainly on partially mixed and well mixed flows, though of course many of their results have significance with respect to highly stratified flows. Still, none of the reports from the West have been of any direct usefulness for solving the problems of highly stratified flows mentioned above. They do not apply to rivers like the Ishikari or Teshio, in which mixing is sometimes so low that they could perhaps be classed as 'very highly stratified' or even 'two-layered' flows.

In contrast, in Japan research into this field was only recently deemed necessary since nearly all the major Japanese rivers have steep gradients and there had been virtually no problems with salt damage inland. Japanese work in this field began from a purely academc viewpoint with Hisao Fukushima's observations of the Ishikari River in 1939 and 1940 (Fukushima, 1942). Those studies were limited in scope to the characteristics of the invading salt water body, or as it is usually called, the salt wedge. After World War II, however, authorities scaled up improvement of river mouths as part of broadened flood control projects. These works brought about sharp alterations in the structure of the salt wedge affecting fisheries and raising the salinity of water withdrawn for agricultural use beyond acceptable levels. Authorities thus became interested in the nature of salt water-fresh water mixing at river mouths; most of the development in this field has come since then. Research in Japan has however, focussed almost exclusively on highly stratified flows.

The most important result in this field has been the derivation of the time average coefficient of friction of the salt wedge. This is found from the following relation, which has been confirmed in nearly all rivers with highly stratified flows:

$$f_i = C(F_i^2 R_e)^n = C\Psi^n . \qquad (1)$$

Here, f_i, F_i and R_e, are, respectively, the coefficient of interfacial friction; the densimetric Froud number, which is calculated using the time mean velocity in the upper (fresh water) layer and the thickness of the upper layer; and the Reynolds number, which is calculated using the velocity and thickness values. Ψ is called the Iwasaki number. The value of coefficient C and n are about 0.25 and -0.5, respectively (Yoshida, 1990).

The physical significance of equation (1) remains unfounded. Little is yet known about the tidal oscillations of the salt wedge level and the process of diffusion of salt into river water. The range of interface oscillation is actually greater than the tidal range, by as much as a factor of seven. This amplitude decreases with distance up the river and is undetectable at the tip of the salt wedge in the Ishikari River, 30 km above the mouth. Therefore, though the condition holds only in the vicinity of the river mouth, the speed of the oscillatory flow of the salt wedge is great enough that it must be taken into account in considerations of speed of the local upper-layer flow. Time-average values of salt wedge velocity do not allow meaningful results. The instantaneous tidal flow velocity is also an essential consideration for stability of the interface.

Research on Mixing Mechanisims at an Interface

The stability of shear flows in the vicinity of an interface and the structure of turbulence which carries salt water into the fresh water layer have proven to be the most important factors influencing mixing across an interface. Theoretical studies of stability in statically stable two-layer flows, show that the instabilities listed in Table 1-A will occur for some forms of the velocity and density distributions. These are the principal instability flows that are known to occur in two-layer flow and have been observed by the researchers listed in Table1-B and others. However, as a result of diffusion and mixing in real stratified flows such as those at river mouths, there often develops a third, long-lived layer of median density between the two main layers. This structure shows instabilities distinct in nature from those of two-layer flows. They are described theoretically by the authors listed in Table 1-C and they have been confirmed experimentally (Caulfield et al., 1994). Yoshida (1977) used the fact that the salt wedge gradually deepens over the distance from its tip to the river mouth to demonstrate the simultaneous existence of K-H instabilities, Holmboe instabilities and Keulegan's 'one-sidedness'. Yoshida (1980) also found instabilities just below the interface which distorted it into a cycloidal-shape. As these instabilities remain even under flow conditions where Holmboe

TABLE 1. A list of selected references on instabilities in layered stably stratified flows.

A	Theoretically Described Shear Flow Instability in Two-Layer Flows
Kelvin-Helmholtz Flow	An explanation of the largest-amplitude instabilities in the interface
Holmboe Flow (1962)	An explanation of the largest-amplitude instabilities occurring simultaneously above and below the interface
Lawrence et al. (1991)	A description of so-called 'one-sidedness'
B	**Experimentally Observed**
Keulegan, G.H. (1949	Observed instabilities just above the interface (correspond to one-sidedness)
Thorpe, S.A. (1968)	Observed Kelvin-Helmholtz instabilities
Browand, F.K. and Wang, Y.H. (1972)	Observed Holmboe instabilities at the shear layer when the ratio of velocity to density length scales was 15
Yoshida, S. (1977)	Observed instabilities occurring independently of each other just above and just the interface when the ratio of velocity to density length scales was much larger than 10
C	**Theoretically Described Shear Flow Instability in Three-Layer Flows**
Taylor, G.I. (1931)	An explanation of the largest-amplitude instabilities within a middle layer of finite thickness
Caulfield, C.P. (1994)	An explanation of instabilities differing from the Holmboe mode which occur just within the upper or lower layers when there is a comparatively thin middle layer

instabilities disappear, it seems implausible to call them a one-sided phenomenon. The mechanism of development can be explained by stability theory on the assumption of a middle layer, however, and some now classify them as one kind of Taylor instability.

Occurrence of these instabilities is to some extent a function of Reynolds number (Nishida and Yoshida, 1983), but at the large scales seen in the field, the velocity and density distributions in the vicinity of the interface can be thought of as dominated by the Richardson number, when properly defined. Determining the Richardson number is the first task when investigating initiation of interfacial instability, the first stage of the transfer of salt across the interface. As mentioned earlier, the flow structure varies continuously from the tip of the salt wedge to the river mouth; thus, the Richardson number varies as a function of location. It is also a function of time in actual flows that are subject to the time-dependent forcing. To give an example, even if the river flowrate is constant the flow is influenced by the tide and so the interface position and shear vary with time. A single measurement of the Richardson number cannot be used to estimate the time average stability of the interface. The Richardson number must be observed over at least one tidal cycle. We emphasize here that even if interfacial instabilities are intense enough to form an intermediate layer, they are not themselves responsible for transport of salt up through the fresh water layer. Transport is brought about by instabilities in the upper layer, or turbulence. Thus, though we refer to this phenomenon as 'transport across the interface', we must go beyond instabilities at the interface to considerations of the turbulence structure, and its time dependencies. In the footsteps of Ellison and Turner (1959), many researchers have examined salt entrainment through turbulence at the interface in the vicinity of the interface; Tamai (1986) and Fernando (1991) have reviewed the most significant recent results. However, all the above researchers focussed on steady turbulent flows, while real flows into the ocean are never steady; it is essential first to identify what effect the factor of unsteadiness has on the structure of very highly stratified flows at river mouths.

Results of Observations in the Ishikari River

The outside factors responsible for time and location dependencies of the Richardson number and time dependencies of upper layer turbulence are not limited to discharge rate and tide. Several other factors such as atmospheric pressure, topography, wind and others play a role and only recently have research efforts been directed into clarifying which were the most influential (Yoshida et al., 1995). Figure 1 shows a map of the lower reaches of the Ishikari River and the points at which measurements were taken in the 1992-1994 Survey. Table 2 provides the distances of each point from the river mouth and the measurements performed there. Figure 2 shows a selection of the long-term results during July and August 1994. The discharge is taken from the river surface level read at St. 15 in the river and a discharge-surface level graph. The discharge was prepared using the relation

$$Q = 79.1\gamma\,(H + 0.11)^2 \text{ for } H = 0.00\text{m} \sim 1.98\text{m and}$$
$$Q = 34.2\,\gamma\,(H + 1.20)^2 \text{ for } H = 1.99\text{m} \sim 6.00\text{m} \qquad (2)$$

where Q is the discharge and H is the level. γ (1.32) is a correction factor to include the influences of the tributaries downstream of St. 15. The overall observation period for the observations listed in the figure was from July 1 to Aug 20, 1994, during the dry period. Two sets of measurements are not listed in Table 2; these were a one-week observation of the velocity at the surface and a 24-hour continuous observation of the height of surface waves,

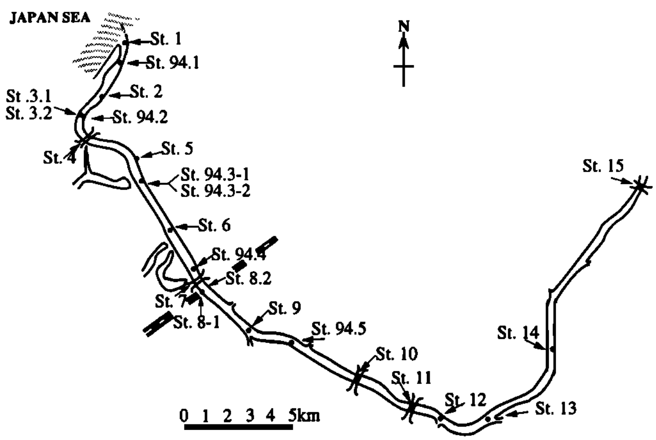

Figure 1. Ishikari River and observation stations.

turbulence and other parameters Aug. 7-8, 1994. Details will be provided during the following discussion as space permits.

Variations in River Surface Level and Salt Transport

The total river depth, one of the most important boundary conditions, is determined by the sea level and the flow rate. Sea level varies with tide and with atmospheric pressure. In order to estimate the latter relation, we used actual sea level data and compared them with the ideal levels predicted by the Meteorological Agency; the deviation is the second time series in Figure 2. This series shows an extremely consistent relation with the atmospheric pressure time series above it in the figure. The quantitative relation obtained is

$$h = a\,(1010 - P) \tag{3}$$

TABLE 2. Location of observations and measured variables on the Ishikari River during 1992-1994.

St.	Dist. from mouth (km)	Variable	St.	Dist. from mouth (km)	Variable
1	0	Interface observation (sonar)	7	14.6	-
94.1	1	Water temp. & Conduct.	8-1	15	Surface level
2	2.9	Water level	8-2	15	Interface level (Step Sensor)
94.2	4.2	Water temp. & Conduct.	9	17.9	-
3-1	4.2	Interface level	94.5	20	Water temp. & Conduct.
3-2	4.2	Surface level	10	23.4	-
4	5.4	-	11	26.6	Surface level
5	8.2	Meteorological data	12	27.9	-
94.3.1	9.3	Assorted data (Platform)	13	30.7	-
94.3.2	9.3	Water temp. & Conduct.	14	35.4	-
6	11.7	Assorted data (Platform)	15	44.6	Surface level
94.4	14	Water temp. & Conduct.			-

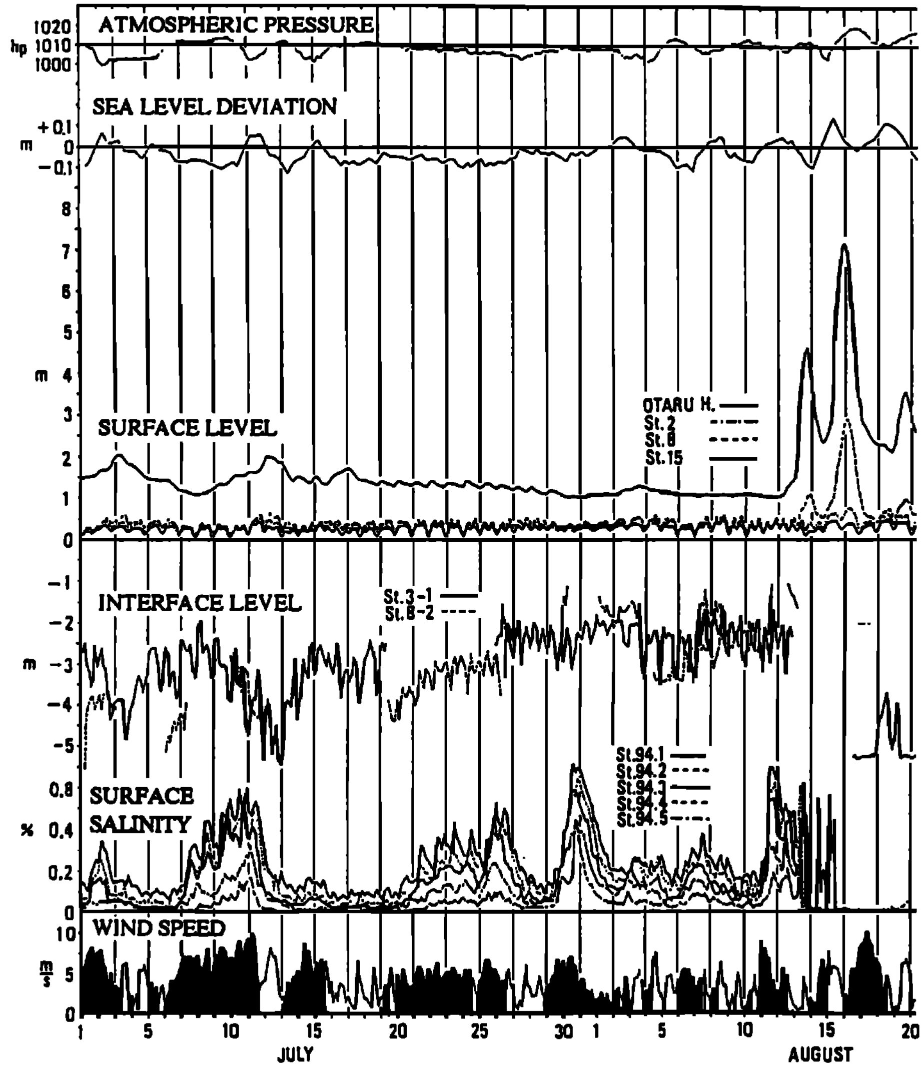

Figure 2. Data during July 1- August 20, 1994 periods

where h is the deviation in cm, a is a value between 1.4 and 1.5 for the Ishikari and P is the atmospheric pressure at the river mouth in hectopascals. Usually, h is a maximum of 20 cm, of the same order as the tidal range in the Japan Sea. Thus, it is necessary to take both tidal range and atmospheric pressure into account. However, the period of the cycle of air pressure change is several times that of the tidal cycle, and atmospheric pressure actually has quite a small effect on river velocity. Even if it acts to raise or lower the river surface, it is a quite minor factor in bringing about unsteady river velocity or salt transport. In comparison, the tide varies on a half-day period and, as mentioned earlier, has a vigorous influence on the two-layer structure and flow velocity, especially during the dry period. The record shows that there was indeed some influence in surface salinity (see Figure 2 record of surface salinity at St. 94.1, 94.2) from the tide, but that effect was limited to the vicinity of the river mouth and to times when the oscillation of the interface was of nearly the same order as the depth of the upper layer; it was virtually never detected at stations upstream of 94.5. From these data, it was clear that other factors than tide were governing the surface salinity.

Wind and Salt Transport

Let us examine the relation between wind speed and surface salinity as revealed in the record. The colored areas in Figure 2 indicate when the wind direction was directly downstream in the straight section of the river channel (from St. 12 to St. 5), and the uncolored areas represent when the wind was in the opposite direction. From these results, we observe that the short-term variations in wind speed make virtually no contribution to variations in the surface salinity, but there is a clear correlation between long-period salinity variations and periods of continued wind. When the data are examined in further detail, the times when the wind began to blow are observed match with the times the surface salinities at all observation points began to increase, but even after the wind velocity fell, the salinity did not fall immediately to the lower, corresponding value, but 'tailed off'. The delay in response increased with distance from the salt wedge tip and was over 1 day at the river mouth. It is impossible to discuss the effect of wind direction since the wind blew almost exclusively in the above two flowing directions in the vicinity of the Ishikari River mouth. However, it is unlikely that winds directions across the river course would bring about higher diffusion rates than winds parallelling the river. The power spectra of the wind speed and salinity support the above results and hypotheses, but cannot be shown here because of space considerations.

We now examine the process of salt transport in order to account for this correlation of wind and surface salinity. As the flow system under consideration is incompressible, both volume and mass are conserved and the following differential equations are assumed to apply.

$$\frac{d(Au)}{dx} = Bv_w \tag{4}$$

$$\frac{d(Au\rho_1)}{dx} = Bv_w\rho_2 \tag{5}$$

Here, A is the cross sectional area of the upper layer of the river, B is the river width, u is the cross-sectional mean velocity of the upper layer, v_w is the velocity of wind-caused entrainment of salt water into the river, x is the coordinate of position measured from the tip of the salt wedge downstream, and ρ_1 is the upper layer density.

The velocity of the lower layer is much smaller than u in this two-layer flow, and B is assumed uniform. Introducing h = A/B and Keulegan's entrainment coefficient $E_w = v_w/u$, (4) and (5) provide the following solution.

$$\frac{(\rho_2 - \rho_1)\big|_{x=x2}}{(\rho_2 - \rho_1)\big|_{x=x1}} = e^{-\int_{x1}^{x2}\frac{E_w}{h}dx} \tag{6}$$

Here ρ is the lower layer density and X_1 and X_2 are the positions of the upstream and downstream end of the river element, respectively. In the Ishikari River the density of the salt wedge has uniform value ρ_2 and generally, salinity S is proportional to the density. Therefore the left hand side of (6) is reduced to

$$\frac{\rho_2-(\rho_1)\big|_{x=x2}}{\rho_2-(\rho_1)\big|_{x=x1}} \cong \frac{S_2-(S_1)\big|_{x=x2}}{S_2-(S_1)\big|_{x=x1}} . \tag{7}$$

The Keulegan coefficient was originally defined for the case where entrainment was considered as dependent only on the difference in the two layers' average velocities; Keulegan had not thought in terms of any effect of the wind. Thus, this differs somewhat in value from the original Keulegan coefficient, which would apply when the wind speed is zero. The average entrainment coefficients in the region between the tip of the salt wedge and St. 94.2 were obtained from the observations as follows:

$$E_w = 1.92 \times 10^{-6}\, e^{0.405\overline{W}} \quad \text{for} \quad 118\text{m}^3/\text{sec} \quad \text{and} \quad 2.5\text{m/sec} \le \overline{W} \le 10\text{m/sec} \tag{8}$$

$$E_w = 2.24 \times 10^{-6}\, e^{0.281\overline{W}} \quad \text{for} \quad 142\text{m}^3/\text{sec} \quad \text{and} \quad 2.5\text{m/sec} \le \overline{W} \le 10\text{m/sec} \tag{9}$$

$\overline{W}$ is the average wind speed during the time necessary for the river water to move from the tip of the salt wedge to St. 94.2 at the cross-sectional average speed. This time would be shorter in case (9) than in (8), but as the difference between the times was small, the average wind speed over a 27-hour period was used. The reason the exponent is higher in (8) is, the flowrate is lower, and the free surface, which is a source of turbulence, is closer to the interface.

Estimate of the Surface Salinity

In light of (8) and (9), we suggest that E_w will be accurately estimated by the following formula for any flow rate and $\overline{W}$:

$$E_w = E(Q)e^{k(Q)\overline{W}} \tag{10}$$

Here, E is the Keulegan entrainment coefficient estimated from the E_w for zero wind velocity, Q is the flow rate, and k is the function of flow rate. E is related to R_i, which is calculated using the depth of the upper layer as the length scale and the difference in velocities between the layers, as

$$kE = CR_i^{-3/2} \tag{11}$$

However, C shows much scatter and it is difficult to choose a credible value. Asaeda and Tamai (1985) explained this scatter by incorporating the Peclet number, which is an index of molecular diffusion. The Peclet number is meaningful in laboratory observations, where wind is not a factor, but in real river mouth flows the effect of wind is large, as shown in (8) and (9), and it is possible that the scatter in C in real flows is due largely to wind.

It is possible to predict the surface salinity of a river at any given location and time from wind data once the variables in (10) are found for the river in question. Figure 3 shows the calculated salinities for the points St. 3-1. The solid line in the graph shows the values found in the survey of 1994, and the dashed line shows the estimates based on the use of (9). To simplify the calculation, the depth of the upper layer and the width of the river were taken to be 3.1 m and 200 m, respectively. The estimated values show acceptable accuracy except for

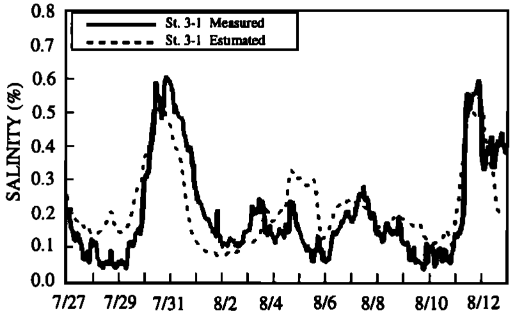

Figure 3. Estimates of surface salinity

the high-frequency variations. The reason that these calculations were of such high accuracy was that both the area of the actual upper layer cross section and the low-frequency component of wind speed were very uniform throughout this 29-km region of the river. As a result, E_w was uniform throughout the region. Accuracy would be improved by incorporating the actual cross sections, local E_w and tidal oscillations of the interface.

Mixing Mechanisms at the Interface

The overturn shown in Figure 4 can always be observed in the dry season just at the outlet of the rivers. The flow velocity and density distribution at the start position of overturn are shown in Figure 5. Because of the deep upper layer, the overturn mentioned above does not occur at the upstream interface of the Ishikari River in any wind conditions. If the wind speed is low, the shear flow is stable and no unstable waves appear at all the interface except at the outlet interface. Figure 6 depicts the distributions of flow speed and density in this low wind speed case where shear reaches a maximum during a tidal cycle. If the distributions of velocity and density Figure 5 are estimated according to the flow model proposed by G.A. Lawrence et al (1991) we obtain Table 3. The length of the instability waves observed in the Ishikari (see Figure 4) conforms closely to the length of the waves predicted by the theory. These results confirmed the validity of stability theory for analysis of real river flows and they have also shown that, rather than Kelvin-Helmholtz waves, one-sided instability waves occur just above the interface. The points of inflection of both curves in Figure 6 almost exactly coincide. Here, the Taylor-Goldstein model, which has linear velocity change and effectively linear density variation, is then used to estimate the Richardson number of this flow. The result (see the right column in Table 3) far exceeds the minimum 0.25 value necessary for stability.

Figure 4. One-sided overturn at the outlet of the river

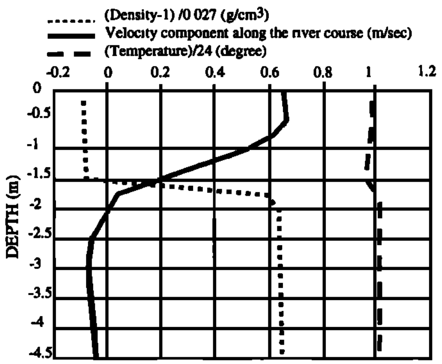

Figure 5. Velocity, density and temperature profile at St. 1 on Aug. 23, 1994

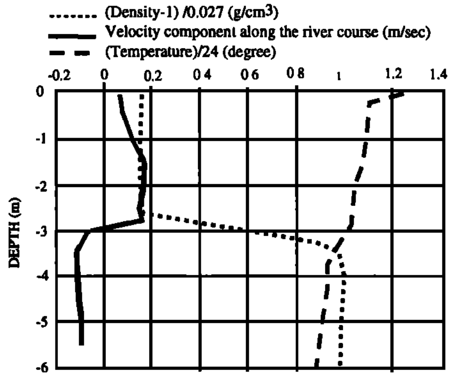

Figure 6. Velocity, density and temperature profile at St. 6 on August 7, 1994

TABLE 3. Measured and calculated values of various quantities relating to shear instability at the mouth of the Ishikari River.

Various Quantities	Measured (St. 1)	Calculated (St. 1)	Measured (St. 1)	Calculated (St. 1)
h: shear depth	1.2m	1.2m	0.73m	0.73m
velocity difference ΔU	0.75m/s	0.75m/s	0.29m/s	0.29m/s
$((\Delta U)$	0.56	0.56	0.08	0.08
wave number k	1.27	1.33	no wave	no wave
$\alpha=kh$	1.52	1.6	no wave	no wave
$\lambda=2\pi/k$	4.96m	4.71m	no wave	no wave
wave speed C	0.4m/s	0.42m/s	no wave	no wave
J: Richardson number	0.56	1.56	2.3	2.3
d: difference between infraction points of U and ρ	0.3m	0.3m	0	0
$\varepsilon=2d/h$	0.5	0.5	0	0

Nonetheless, the above considerations apply only when the effect of wind can be ignored. When the wind blows, gravity waves often arise to destroy the interface. The details of the complicated mechanisms of this transformation remain undescribed, however, as do those responsible for transport of salt to the surface once the interface is gone. It is necessary to pursue the study of the mechanisms for instability at the interface and in the upper layer as well as the structure of turbulence. Little is yet known of the structure of turbulence. Much statical evidence has been gathered on the relation between wind and turbulent intensity, but the Reynolds stress in real cases is too low for any correlation to be credible. Figures 7 and 8 show the horizontal and vertical components of the turbulent intensity with respect to wind speed. The buoyancy flux, which includes the vertical component of turbulence, probably stated as a function of wind speed, but the presently available data on density fluctuations are unreliable. Measurements at two vertical locations have reliably shown, however, that as wind speed increases, the vertical density gradient decreases to uniform salinity. This is clear evidence that salt transport out of the lower layer increases with wind speed (Yoshida, 1995).

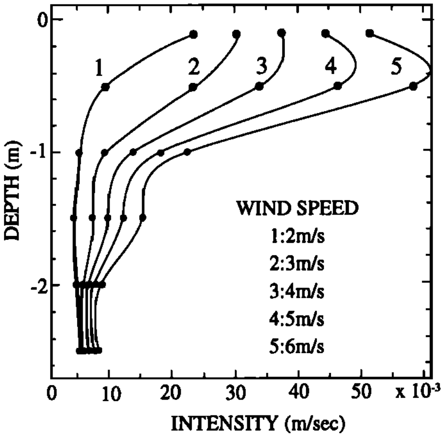

Figure 7. Horizontal turbulence intensity at St. 6.

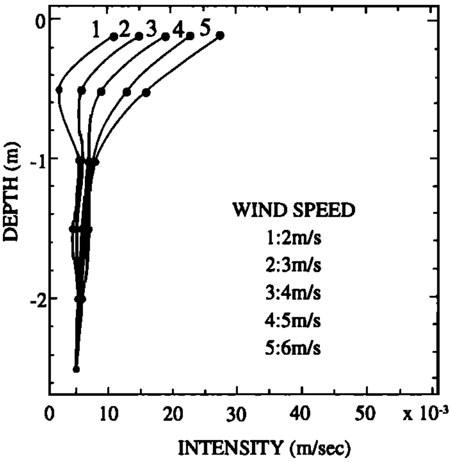

Figure 8. Vertical turbulence intensity at St. 6

Conclusions

This paper describes measurements on a salt wedge in a strongly stratified river. Stability of the flow in the Ishikari River can be determined using stability parameters from the Taylor Goldstein model. The entrainment of salt into the upper layer is determined by both the wind speed and shear velocity between the two layers.

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