J. Fluid Mech. (1973), *vol.* 60, *part* 3, *pp.* 467–480
Printed in Great Britain

The interaction of a vortex ring with a sharp density interface: a model for turbulent entrainment

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(Received 11 July 1972 and in revised form 30 May 1973)

The interaction of a vortex ring with a sharp density interface is investigated in the laboratory. Attention is restricted to the case where the Froude number based on the density difference across the interface, the velocity of propagation of the ring normal to the interface and the diameter of the ring is less than unity. It is found that the depth of maximum penetration of the ring, and the diameter of the region of contact between the ring and the interface, are functions of the Froude number. A simple model of the ring–interface interaction which accounts for the observed motion is proposed. This model is then used to calculate the volume rate of entrainment produced by the vortex rings. It is found that this rate of entrainment is proportional to the cube of the Froude number, a result which agrees with measurements of entrainment across density interfaces caused by grid-generated turbulence (Turner 1968) and by a plume incident on the interface (Baines 1973). Thus the vortex ring would appear to be a good approximation to a turbulent eddy in these situations. The main feature of the model is that it identifies the way in which the kinetic energy of the turbulence is converted into potential energy by entraining fluid across the interface. In particular, it indicates that the essential force balance is inertial, and that it is possible to discuss entrainment across a sharp density interface without explicitly invoking either viscosity or molecular diffusion.

1. Introduction

In many geophysical situations sharp density interfaces are found to exist between two relatively uniform layers. Examples of naturally occurring interfaces across which the rates of entrainment are small and where the turbulent motions and density steps are set by external features are found in both the atmosphere and the ocean. Turner & Kraus (1967) discuss the seasonal thermocline as an interface between a wind-stirred warm surface layer and non-turbulent cooler deep water. Other examples of sharp density interfaces are found in the oceanic microstructure and in the subsidence inversion in the atmosphere. Experiments with inclined plumes, of the kind performed by Ellison & Turner (1959), have shown that the vertical transport of heat, solutes and momentum is controlled by the transport across these stable interfaces. The rate of mixing across an atmospheric inversion is a controlling factor in the removal of pollutants from the immediate vicinity of urban and industrial areas. It is therefore desirable to have an understanding of the detailed mechanisms of turbulent entrainment across sharp density interfaces.

A first step towards such an understanding has been to examine the mixing across an interface produced by turbulence which is externally imposed and controlled. The initial investigation was made by Rouse & Dodu (1955), who measured the entrainment caused by turbulence produced by an oscillating screen. This work has been followed up more recently with experimental studies by Turner (1968), Thompson (1969) and Baines (1973). A brief review of their results will be given in order to motivate the work reported in this paper.

Turner measured the rate at which fluid is entrained across an interface when the turbulence is produced by an oscillating grid. (For details of the experimental apparatus the reader is referred to the original paper.) A level grid generates a horizontally homogeneous layer of turbulent fluid; fluid is then entrained from the non-turbulent region to the turbulent region until the turbulence fills the tank. The sharp interface in this situation is characterized by a thin region within which there are large gradients of velocity and density. Turner considered two situations: in the first case only one grid was used and the interface held stationary by removing fluid from the turbulent region. In the second case two grids were used and the interface then remained stationary midway between the grids. The rate at which fluid was entrained across the interface was measured as a velocity u_e, defined as the amount of fluid entrained per unit area per unit time. Turner found that u_e was a function of an interfacial Froude number defined by

$$Fr^* = n(\rho_0 \hat{l}/g\Delta\rho)^{\frac{1}{2}}, \tag{1.1}$$

where n is the frequency of oscillation of the grid, $\Delta\rho$ is the density difference across the interface, ρ_0 is the mean density, g is the acceleration due to gravity and $\hat{l}$ is an arbitrary length scale.

Thompson (1969) measured the r.m.s. horizontal velocity u and the integral length scale l of the turbulence produced by oscillating grids in a homogeneous fluid. These measurements, made with a hot-film probe, included some for the grids used by Turner and were carried out in the same tank. It is thus possible to use Thompson's measurements to re-evaluate Turner's results. It was found that at a fixed distance from the grid, $u \propto n$ whilst l was independent of n (for a fixed amplitude of oscillation). Thus, identifying $\hat{l}$ with the integral scale of the turbulence and replacing n by the velocity scale u, Turner's results can be expressed in terms of the flow variables at the interface, which give a non-dimensional Froude number of the form

$$Fr = u/(lg\Delta\rho/\rho_0)^{\frac{1}{2}}. \tag{1.2}$$

Turner's measurements of the entrainment rate for both the one- and two-grid cases are consistent with the power laws

$$u_e/u \propto Fr^2, \tag{1.3}$$

when the density step was produced by a temperature difference, and

$$u_e/u \propto Fr^3, \tag{1.4}$$

when the lower layer was made more dense by the addition of salt.

It was pointed out by Rouse & Dodu (1955) that (1.3) implies a rate of change of potential energy due to mixing which is proportional to the rate of production of kinetic energy by the grid. It is therefore appealing to explain the heat transport on the basis of this energy consideration and then examine various hypotheses for the reduced salt transport. This type of argument was made by Turner (1968) but it will be seen that in fact (1.4) is the fundamental mixing rate and may be explained in terms of an inertial entrainment process.

An indication that (1.4) is the basic relation has come from some experiments by Baines (1973) in which he measured the entrainment across a salt interface produced by a turbulent plume incident normally on the interface. The rate of entrainment u_e across the interface was estimated from the change in the position of the interface. Baines found that his results were consistent with (1.4) when u and l were appropriately defined as the velocity and length scales associated with the plume motions at the interface. Further evidence of the fundamental nature of (1.4) has been provided by C. Rooth (private communication), who has found that the heat transport produced by stirring grids is consistent with (1.4) when the Péclet number $Pe = ul/\kappa$ of the turbulent field is sufficiently large.

Thus the experiments of Baines and Turner suggest that two apparently different forms of turbulent motion produce the same rates of entrainment across a density interface. In fact not only is the power-law dependence on the Froude number the same but also the numerical values for the entrainment rate for each process are similar (see figure 7).

In this paper some experiments are described in which the detailed mechanisms producing mixing when a vortex ring is incident normally on a sharp density interface are examined. As for the experiments described above it is possible to set the velocity u and the length scale l of the vortex ring and the density difference $\Delta\rho$ across the interface independently. Therefore, this investigation excludes such flows as jets, plumes, wakes and shear layers, where the entrainment, either across a density interface or to a turbulent from a non-turbulent region, is a controlling feature of the dynamics of the flow. In the following sections the experiments and the experimental results are described. These results are then examined on the basis of a simple model for the ring–interface interaction. The results of the ring–interface interaction are then carried over to the entrainment by the plume and grid-generated motions by the assumption that a spherical vortex is a good approximation to the energy-containing eddies in the turbulent fields. It will be seen that the mixing rate may be explained in terms of inertial dynamics, thereby avoiding the explicit use of viscosity or diffusion in the entrainment process.

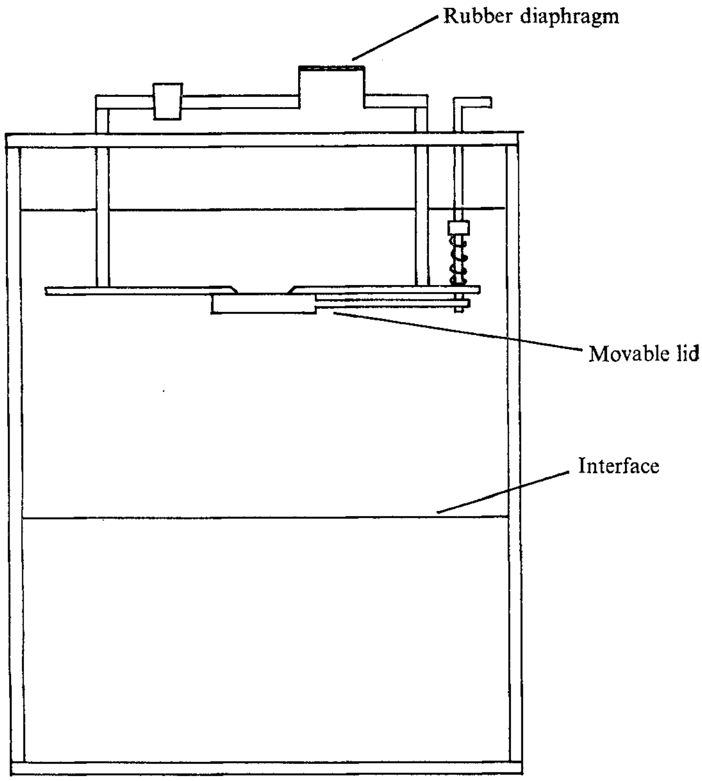

FIGURE 1. A sketch of the experimental tank and the chamber used to produce the vortex rings.

2. The experimental method

The experiments were carried out in a rectangular perspex tank 40 cm deep and 25×15 cm in cross-section. At the top of the tank a chamber, designed for the production of vortex rings, was inserted into the water as shown on figure 1. The chamber had a circular hole in the centre of its base plate and a movable lid was fitted over this hole. A rubber diaphragm was stretched over a second hole in the roof of the chamber. The vortex rings were formed by striking this diaphragm; this action imparts an impulse to the fluid in the chamber and causes some of this fluid to move through the hole in the bottom of the chamber thereby producing a vortex ring which propagates down through the depth of the tank. (This method is a standard way of generating vortex rings in the laboratory; for a detailed account of the formation of a ring by this method see Maxworthy (1972).)

The tank was filled with two layers of water, the lower being made more dense by the addition of salt. In order to obtain a sharp interface between the layers the more saline layer was carefully added beneath the fresh layer. The interface was sharpened by siphoning fluid from the centre of the interface. It was found that the interface could be made very sharp by this method and in every run the thickness of the interface was always much smaller than the diameter of the vortex ring and never exceeded 0·2 cm. The densities of the fluid in the layers were determined by an hydrometer and were known to an accuracy of 0·001 g ml^{-1}.

The vortex rings were fired individually at the interface and the resulting motions were photographed by a movie camera. The flow was visualized by dyeing the water in the chamber, and therefore the core of the vortex ring, and viewing the interaction of the ring and the interface by means of a shadowgraph. The fluid in the chamber was of the same density as the fluid in the upper layer and the movable lid was used to contain the dye in the chamber before and after the ring was fired.

From a frame-by-frame examination of the movie films the velocity of propagation u and diameter l of the rings were measured as well as the distortions of the ring and the interface during impact. Care was taken to ensure that the measurements of the velocity and diameter of the rings were made before the interface had any noticeable effect on the motion of the rings. However, Maxworthy (1972) has shown that for laminar vortex rings the velocity of propagation falls off exponentially with the distance from some virtual origin. Hence it was necessary to measure u and l as close as possible to the interface.

The vortex rings produced by the method described above were related in size to the diameter of the hole in the base plate of the chamber. Two hole sizes were used (1 cm and 2 cm in diameter) and the rings generated were between 1 cm and 4 cm in diameter. The velocity of propagation of the rings was of the order of 2 cm s^{-1}. Seven density steps were used with values of $\Delta\rho/\rho_0$ ranging from 0·006 to 0·067. The Froude number of the flow based on the density difference between the layers, the velocity of the ring perpendicular to the interface and the diameter of the ring varied between 0·14 and 0·80.

3. The experimental results

Qualitative results

The flow produced by the impact of a vortex ring with a sharp interface is shown on figure 2 (plate 1). Note the similarity of the distortions of the interface to those produced by the action of grid-generated turbulence as shown by Turner (1968). It is convenient to divide the interaction into two parts, an 'impact stage' and a 'recoil stage'. The impact stage is defined as the time between the initial moment of interaction with the interface and the time at which the velocity of the ring in the direction normal to the interface is zero. The motion during this stage is shown on figure 2 (*a*); the interface is deflected downwards by the ring, which itself is flattened, retaining circular symmetry about a vertical axis through the centre of the ring. In the case when the Froude number is small ($\lesssim$ 0·1) the interface tends to behave like a rigid wall, the radius of the ring increasing with decreasing distance from the interface. As the Froude number increases, the deflexions of the interface increase and the spreading of the ring is reduced (see figures 4 and 5).

The motion during the recoil stage is shown on figure 2 (*b*); after the interface reaches its points of maximum deflexion, the buoyancy forces cause the interface

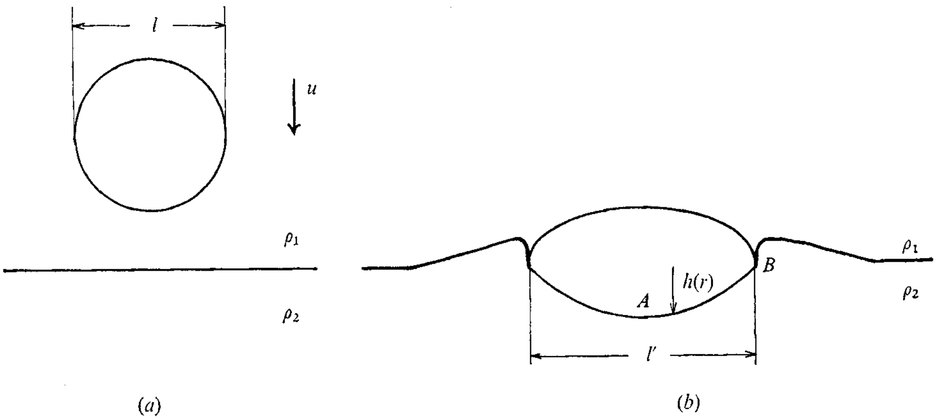

FIGURE 3. A diagrammatic representation of the ring-interface interaction.

to recoil, the ring to collapse and denser fluid to be ejected into the upper layer. This denser fluid then mixes with the fluid in the ring and the vorticity produced by the recoil of the interface cancels the vorticity in the ring.

The mixing appears to occur only at the initial recoil of the interface. Estimates taken from the movie of the volume of mixed fluid indicate that this remains constant after the initial collapse of the ring. The region of intermediate-density fluid then spreads horizontally along the interface without a significant change in volume. When other turbulent motions are present this mixed fluid will be carried away and mixed through the whole of the upper layer. It was also observed on the shadowgraph that fluid was lifted around the outside of the ring before the recoil, presumably by the action of viscous stresses across a thin boundary layer between the ring and the lower layer. It will be shown that for the range of Reynolds and Froude numbers considered in these experiments, $360 \leqslant Re \leqslant 1080$ and $0{\cdot}14 \leqslant Fr \leqslant 0{\cdot}8$, the torque produced by this viscous entrainment is small compared with that produced by the tilt of the interface and the entrainment rate may be evaluated without invoking viscosity.

Quantitative results

The impact of a vortex ring with an interface is depicted diagrammatically in figure 3. The features of the impact, namely the profile $h(r)$ of the distorted interface, the diameter l' of the region of contact of the ring and the interface and the time scale t_B of the recoil, were measured from the movie. $h(r)$ and l' were determined at the instant when the velocity of the ring normal to the interface was reduced to zero by the action of the buoyancy forces. t_B was taken to be the time elapsed from the moment of maximum distortion of the interface until the fluid was ejected into the upper layer (see figure 2(*b*)).

It was found that the shape of the distorted interface varied with the Froude number. The profiles were of the form

$$h(r) = h_m[1-(2r/l')^n], \tag{3.1}$$

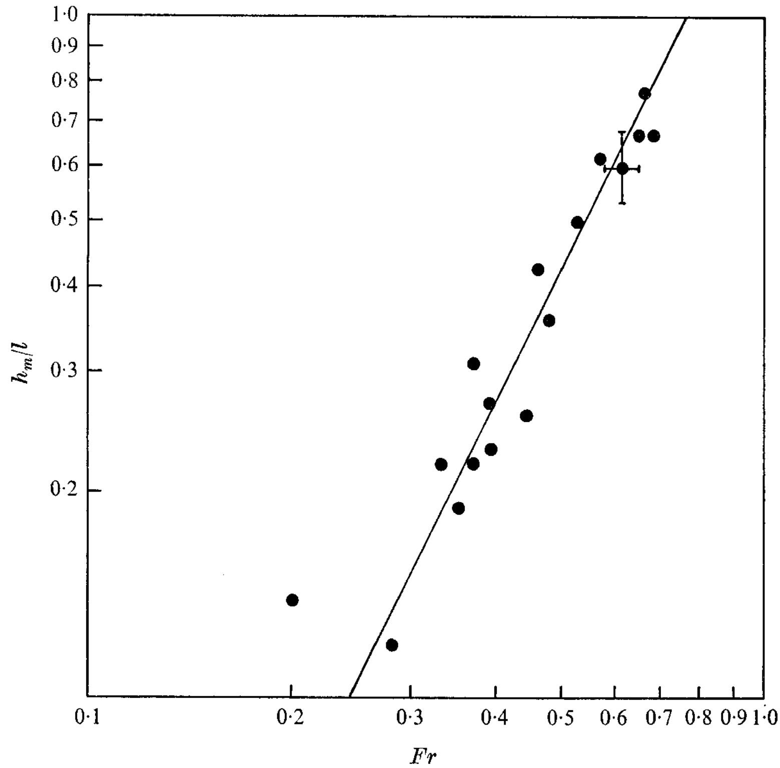

FIGURE 4. A log–log plot of the depth h_m of maximum penetration of the ring against the Froude number. The solid line shows the best fit to the data with slope 2.

where $n \approx 2$ for $Fr \approx 0{\cdot}8$, and n increases as the Froude number decreases with $h(r)$ becoming approximately constant over the region of contact between the ring and interface for $Fr \leqslant 0{\cdot}3$. However, the measurements were not sufficiently accurate to determine any functional dependence of n on Fr.

The results of the measurements of h_m, l' and t_B are shown on figures 4, 5 and 6, respectively. These data will not be discussed here; a simple model of the ring–interface interaction will be presented first and the results discussed in the light of this model.

4. A theoretical model

The experimental results described in the previous section will now be discussed in terms of a simple model. This model, which considers the interaction of an initially spherical vortex and a density interface, is used to obtain an estimate of the entrainment rate when the turbulent motions can be represented by an ensemble of eddies (taken here to be spherical vortices).

Consider the situation shown on figure 3 (*b*); at this instant in time the velocity of the vortex ring in the direction normal to the interface is zero, although there is still rotational motion in the interior of the ring. The application of Bernoulli's

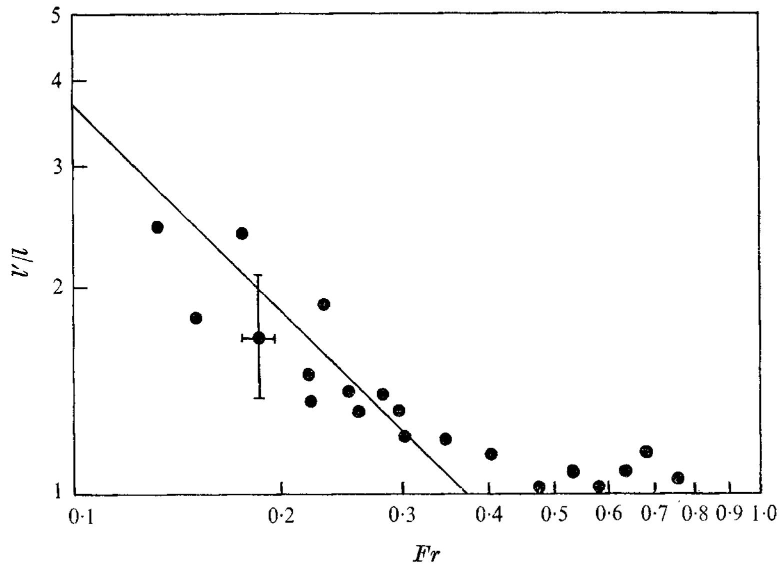

FIGURE 5. The diameter of the region of contact between the ring and the interface l' plotted against the Froude number. The solid line has a slope of -1.

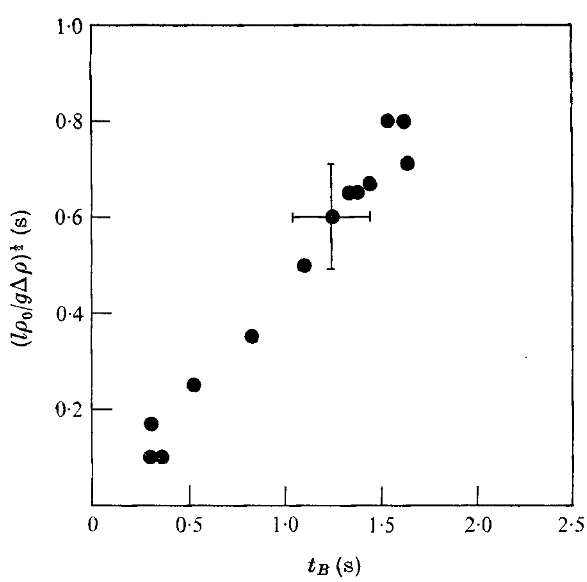

FIGURE 6. The time scale t_B for the recoil of the interface plotted against the Brunt–Väisälä time $(l\rho_0/g\Delta\rho)^{\frac{1}{2}}$ for the response of a density interface to a disturbance of scale l.

equation along the streamline passing through the stagnation point A and the point B gives

$$g\Delta\rho h_m = \tfrac{1}{2}\rho_0 U^2 - \rho_0 \dot{\phi},$$

where U is the speed at B of the fluid along the streamline and ϕ the velocity potential. In order to obtain an estimate for $\dot{\phi}$ note that

$$\dot{\phi} = \int \dot{w}\, dz = \int \dot{w} w\, dt = \tfrac{1}{2}w^2.$$

The motion of the interface during the impact is produced by the forward motion of the ring as a whole. In the case of a spherical vortex the velocity of this translational motion $u \propto U$. Hence, assuming that both w and U are proportional to the velocity of translation of the vortex ring, substitution into Bernoulli's equation gives

$$h_m/l \propto Fr^2. \tag{4.1}$$

This relationship is seen to be consistent with the data shown on figure 4, where the solid line drawn through the data points has a slope of 2. The experimental data show that the constant of proportionality in (4.1) is $c_1 = 1{\cdot}72 \pm 0{\cdot}32$.

The buoyancy forces produced by the distorted interface arrest the translational motion of the ring, and it is assumed that the kinetic energy of this translational motion is converted into potential energy temporarily stored in the distorted interface. For a spherical ring of diameter l the kinetic energy of its translational motion is proportional to $\rho_0 l^3 u^2$. The potential energy of the distorted interface (at its point of maximum distortion) is proportional to

$$[n^2/(n+1)(n+2)]\, g\Delta\rho l'^2 h_m^2,$$

where n is the exponent in (3.1). The equality of these two energies implies that

$$\begin{aligned} l'/l &\propto [2(n+1)(n+2)/3n^2]^{\frac{1}{2}}\, Fr^{-1} \\ &= c_2(n)\, Fr^{-1}. \end{aligned} \tag{4.2}$$

Note that the kinetic energy associated with the rotational motion of the ring is not included in the above energy balance.

In the previous section it was noted that, for $Fr \lesssim 0{\cdot}3$, $h(r)$ is approximately constant over most of the region of contact between the ring and the interface. In this limit ($n \to \infty$) the data do appear to be consistent with (4.2) with $c_2 = 0{\cdot}37 \pm 0{\cdot}11$ (see figure 5). At larger values of Fr, n varies with Fr and the simple functional dependence of l'/l on Fr described by (4.2) no longer strictly applies. However, in the remainder of this section it is assumed that (4.2) is valid (with c_2 constant). This assumption breaks down as $Fr \to 1$, when the interface is relatively unstable, and this situation is excluded from the following discussion. Note further that (4.1) and (4.2) imply that $l'^2 h_m \propto l^3$, consistent with the observation that no mixing occurs during the impact stage.

The consequences of the model will now be extended in order to obtain a prediction for the rate of entrainment of fluid across a stable density interface when turbulent motions are imposed upon it. It is assumed that the turbulent motions may be considered as consisting of a number of discrete energy-containing eddies which interact with the interface in a way similar to that of the vortex rings described above. (Turner's (1968) photographs certainly indicate the existence of many such discrete entities.) Let u and l be typical velocity and length scales of the turbulence in the vicinity of the interface. It is envisaged then that a turbulent eddy will come into contact with the interface, its forward motion will be arrested by the buoyancy forces and then fluid will be ejected across the interface by the recoil of the interface.

The energy supply for this entrainment mechanism comes from the kinetic energy of the turbulent motions which is stored temporarily as potential energy in the distorted interface. Suppose that, on the average, there are N eddies interacting with the interface at any one time: then the energy stored in the interfacial distortions is proportional to $N\rho_0 l^3 u^2$. This energy is released by the recoil of these distortions; therefore, the rate at which energy is made available for the entrainment per unit *active* area is proportional to $N\rho_0 l^3 u^2/\tau N l'^2$, where τ is the time scale of the recoil. The rate at which the potential energy of the system (per unit area) is increased is proportional to $g\Delta\rho u_e d$, where d is the height to which fluid is raised by the recoil.

It is necessary now to obtain expressions for the height d and the time scale τ of the recoil. As the length scale of the turbulent motions is l it is necessary that $d \gtrsim l$ in order that the mixed fluid be made available to the turbulent motions not in contact with the interface. As the interface was observed to remain sharp in Turner's (1968) experiments with salt, and in the case of the vortex rings the mixing occurred on a scale comparable with the ring size it is assumed that $d \approx l$. The time scale τ results from the reaction of the interface to a disturbance of scale l. Therefore, taking τ to be the natural response time of an interface to a forcing of scale l, one has $\tau \propto (\rho_0 l/g\Delta\rho)^{\frac{1}{2}}$.† This is in agreement with the observed recoil time of the interface t_B shown on figure 6; the data are consistent with

$$t_B = c_3(\rho_0 l/g\Delta\rho)^{\frac{1}{2}}, \tag{4.3}$$

where $c_3 = 2{\cdot}28 \pm 0{\cdot}41$. These expressions for d and τ are mutually consistent in the following way. The vertical force (produced by the interface) on the ring is proportional to $g\Delta\rho h_m l'^2 \propto g\Delta\rho l^3$. As the mass of the ring is proportional to $\rho_0 l^3$, the vertical acceleration of the ring is $g\Delta\rho/\rho_0$. Consequently, the height to which the interface can rise under this acceleration for a time τ is

$$d \propto g(\Delta\rho/\rho_0)\,\tau^2 \propto l.$$

Hence if the rate of kinetic energy released is equated to the rate of increase of potential energy produced by raising fluid across the interface, the non-dimensional entrainment rate is given by

$$\begin{aligned}\frac{u_e}{u} &\propto Q\frac{u^2}{g\Delta\rho l/\rho_0\, l}\frac{l^2}{l'^2}\frac{l}{ut_B}\\ &= \gamma Q Fr^3,\end{aligned} \tag{4.4}$$

† A derivation of this time scale in terms of a wavelike response of the interface has been pointed out to the author by Dr E. J. Hinch. The impact of the ring of diameter l excites similarly scaled wave components which have a velocity $c = (g\Delta\rho l/\rho_0)^{\frac{1}{2}}$. If the ejection of fluid into the upper layer is caused by these waves breaking then the interface recoil has a time scale of these principal waves of length l; i.e. $\tau \propto (\rho_0 l/g\Delta\rho)^{\frac{1}{2}}$.

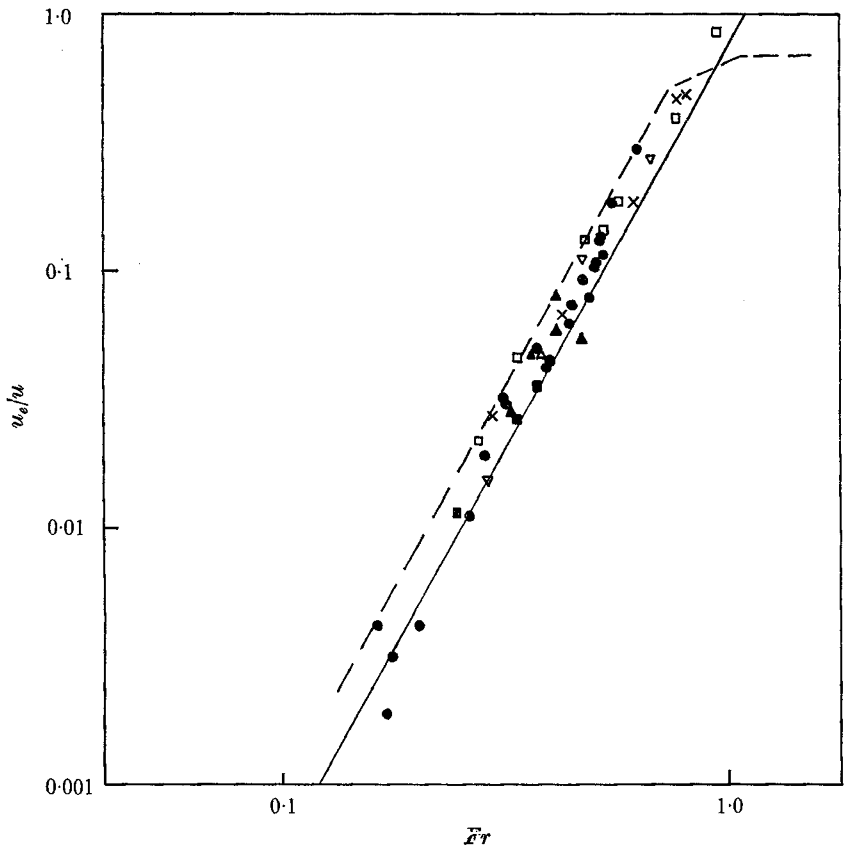

FIGURE 7. The entrainment across a density interface by a plume as measured by Baines (1973) as a function of the Froude number based on length and velocity scales associated with the plume motions. – –, mean of Turner's (1968) measurements of entrainment across a salt interface caused by grid-generated turbulence; ——, equation (4.4) with $\gamma = 0{\cdot}6$ and $Q = 1$.

where γ is a constant determined by the c_i ($i = 1, 2, 3$) and Q is the ratio of the active area of the interface to its total area. Using the experimentally determined values of the constants in (4.1)–(4.3) one finds that $\gamma = 0{\cdot}6 \pm 0{\cdot}2$.

The data shown on figure 7 are the non-dimensional entrainment rates produced by a plume incident on the interface as measured by Baines (1973). The broken curve shows a summary of Turner's (1968) measurements for entrainment across a salt interface produced by grid-generated turbulence. The full curve is a plot of (4.4) with $Q = 1$ and $\gamma = 0{\cdot}6$. The agreement with the experimental data is satisfactory although it is to be expected that in general $Q \leqslant 1$; however, the value of γ determined from these experiments with ring vortices will not strictly apply to these other turbulent situations. The main point to notice is the agreement in the functional dependence of the Froude number. The value of Q depends on the detailed way in which the eddies are distributed in the flow; the model presented above leaves Q undetermined.

5. Discussion

In the case of grid-generated turbulence it was found (Turner 1968) that the constants of proportionality in (1.3) and (1.4) were independent of whether one or two grids were used. This fact indicates that the turbulent motions on the two sides of the interface were decoupled in the sense that the arrival of two eddies at the interface at the same time and in the same position was a rare occurrence. Furthermore, Thompson's (1969) measurements of the r.m.s. horizontal velocity were horizontal averages at given distances from the grids. Therefore, as this average is made up of a series of separate turbulent elements (modelled by the rings) acting over the total area of the interface, it would seem reasonable that a single-element model would provide a realistic prediction of the entrainment rate. The assumption that a vortex ring acts as an 'idealized eddy' appears to be quite plausible in Turner's experimental arrangement. In his case the eddies were self-propagating (there was no significant mean flow in the tank); for an energy-containing eddy to remain a definite entity its velocity of propagation would have to be related to its vorticity structure in a manner similar to that for a spherical vortex (i.e. $u \sim \omega l$).

In the case of a plume incident on the interface providing the response time $O(l\rho_0/g\Delta\rho)^{\frac{1}{2}}$ of the interface is shorter than the arrival time $O(l/u)$ of the eddies, the interface will react to the plume as though it consists of a series of discrete eddies. This restriction requires that $Fr < 1$, which was the case in Baines' experiments. Observations of the entrainment process reported by Baines (private communication) indicate that the more dense fluid was lifted into the upper layer in the form of wisps or streamers. The majority of these streamers appeared to be concentrated towards the centre of the depression of the interface. Further, the distorted interface was not steady but moved in a complicated three-dimensional manner. These observations are consistent with the idea that the heavier fluid was ejected into the upper layer by the interface recoil.

In many geophysical flows it is of interest to specify the entrainment in terms of 'overall' parameters. For example, in the case of the flow of a plume along a sloping boundary it is desirable to be able to relate the entrainment (and therefore the spread of the plume, etc.) to the source conditions of the plume and the slope of the boundary. An attempt at this type of problem has been made by Kato & Phillips (1969), who measured the entrainment across an interface when the turbulence was produced by a shear stress applied to the surface of a linearly stratified fluid layer. They found that the dimensionless entrainment rate was a function of an overall Richardson number defined by $Ri_0 = g(\partial\rho/\partial z)_0 D^2/2\rho_0 u_*^2$, where D is the depth of the layer, $(\partial\rho/\partial z)_0$ the initial density gradient and u_* is the friction velocity associated with the stress applied at the upper boundary. Unfortunately, their results have a large amount of scatter and the authors claim, on the basis of Turner's (1968) analysis, that the data are consistent with $u_e/u_* \propto Ri_0^{-1}$. However, their published data are also consistent with the relation $u_e/u_* \propto Ri_0^{-\frac{3}{2}}$, and in fact the latter relation is a better fit for some of the individual runs. Recently, Moore & Long (1971) have studied the entrainment across an interface between two layers of fluid moving in opposite directions around an annular tank. Their results also indicate that $u_e/u \propto Ri_0^{-1}$, where Ri_0 is an overall Richardson number appropriate to their experimental configuration. However, the method of production of the mean flow in the tank (by the use of jets of fluid at the top and bottom boundaries) must introduce turbulent motions on scales quite different from those produced by the mean flow which cannot be accounted for in their definition of Ri_0. It is interesting to note that $u_e/u \propto Ri_0^{-1}$ (which is equivalent to (1.3)) implies that the density difference across the interface does not enter into a determination of the buoyancy flux. For large values of $\Delta\rho$, u_e is small and as $\Delta\rho$ decreases u_e increases so that the buoyancy flux remains constant.

The effect of viscosity

It was noted that as well as by ejection due to the recoil of the interface fluid was lifted from the lower layer around the edge of the ring. This fluid is presumably raised by the action of viscous stresses across a thin boundary layer. The more dense fluid in this boundary layer exerts a torque on the ring which opposes the circulation in the interior of the ring. If the boundary-layer thickness is of order $(\nu t_B)^{\frac{1}{2}}$ then it is easy to show that the ratio of this viscous torque to the torque produced by the tilt of the interface is $Re^{-\frac{1}{2}}Fr^{-\frac{3}{2}}$, where $Re = ul/\nu$ is the Reynolds number of the ring. For all the results presented in § 3 the value of this ratio is less than 0·1. When the ratio was $O(1)$ it was found that the dramatic collapse of the ring due to the recoil of the interface did not occur and the ring maintained its shape for much longer.

6. Summary

A model for turbulent entrainment across a stable density interface has been presented on the basis of the observed interaction of a vortex ring and a density interface. The measured rates of entrainment produced by grid-generated turbulence and by a plume incident on an interface have been explained in terms of inertial processes. It has been demonstrated that that rate of entrainment u_e produced by high Reynolds number, high Péclet number flow is related to the Froude number Fr by a relation of the form $u_e/u = \gamma Q\, Fr$, where γ is a constant depending on the exact form of the turbulent eddies and Q is the ratio of the area of the interface at which entrainment is taking place at any one time to the total area.

Although the ring–interface model does not apply directly to situations where the turbulent eddies are advected along by a mean flow or where the density gradients affect the turbulence, the experiments of Kato & Phillips (1969) give some support to the idea that (4.4) describes the entrainment rate even in these dynamically more complicated flows. In such situations the numerical values of γ and Q would incorporate the details of the relationship between the overall parameters and the resulting velocity and density structure of the flow.

I wish to thank Dr T. Maxworthy and Dr J. S. Turner for many helpful discussions on the work described above. This work was started whilst I was a student at the Geophysical Fluid Dynamics Summer Program at Woods Hole in 1970, and was supported by a grant from the National Science Foundation at that time. The completion of the work was supported by a grant from the Natural Environment Research Council.

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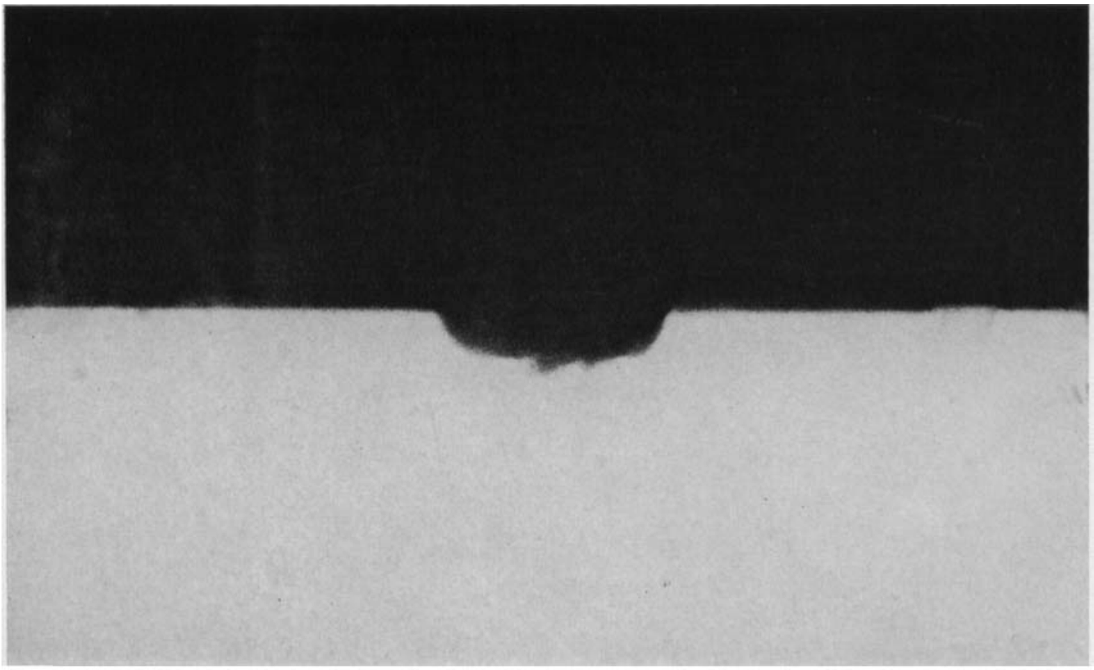
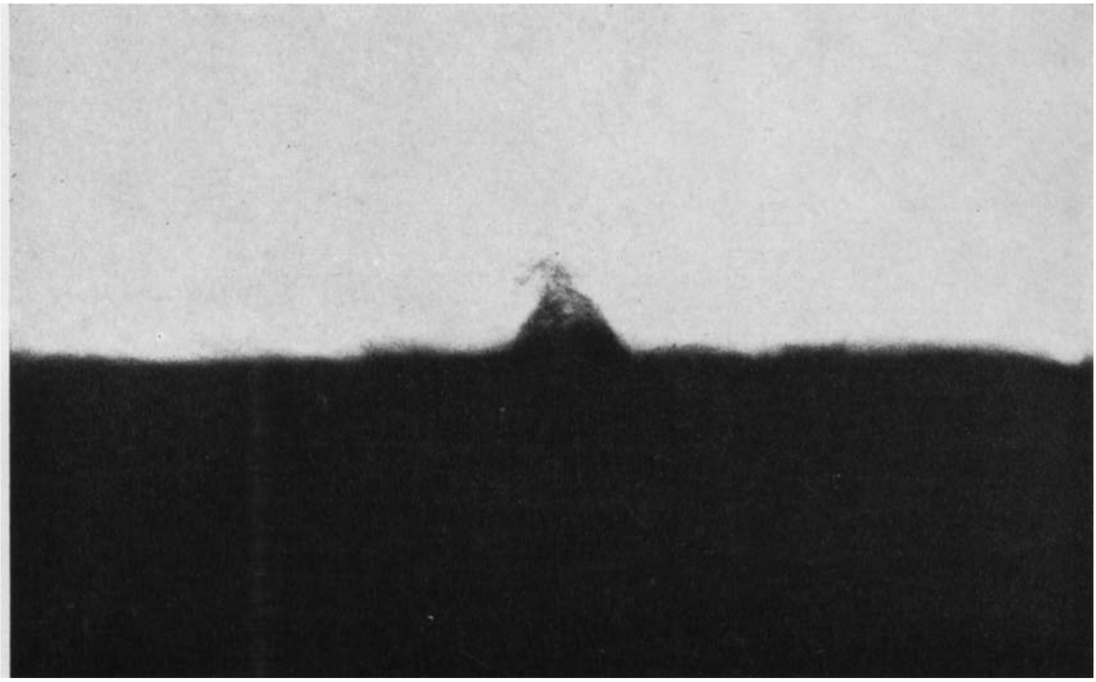


(*a*)



(*b*)

FIGURE 2. The distortions of a density interface caused by the impact of a vortex ring. In both cases the ring is fired downwards onto the interface. (*a*) The penetration of the vortex ring (dyed) into the lower layer. (*b*) The ejection of the denser fluid into the upper layer caused by the recoil of the interface.